Dynamics and Control of Formation Flying Satellites

NASA Lunch & Learn Talk

August 19, 2003

by

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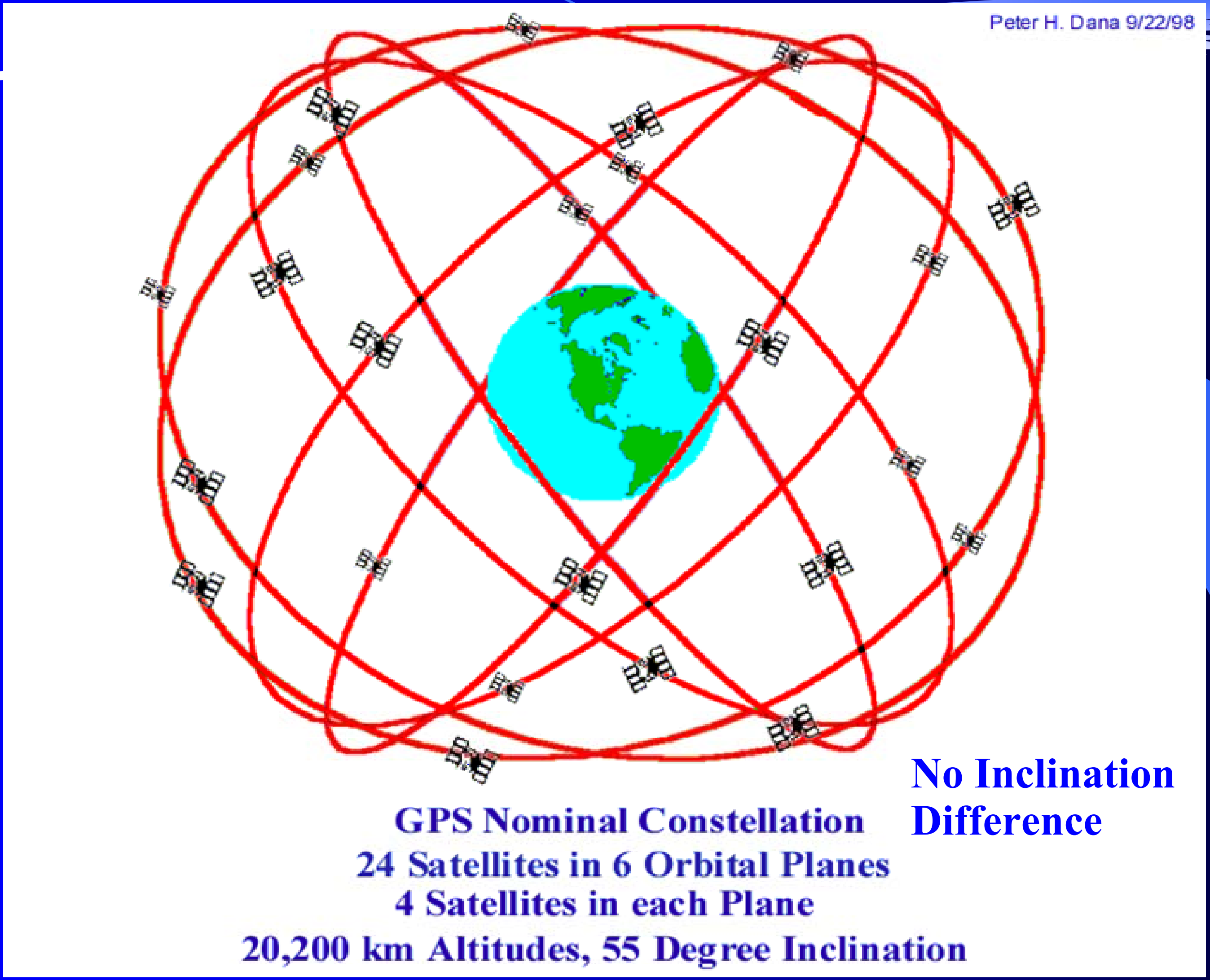
Outline

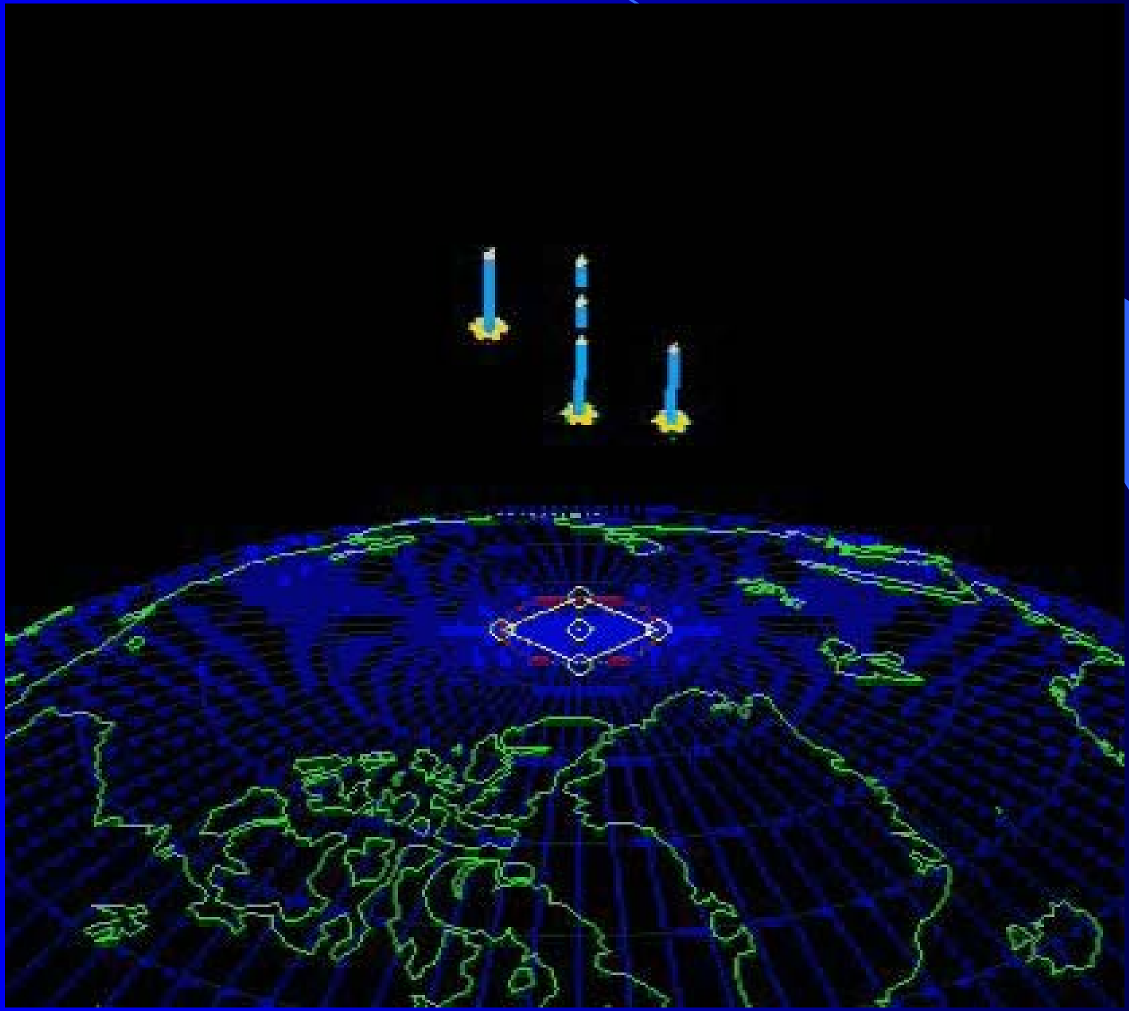
- Formation vs. Constellation
- Introduction to Orbital Mechanics
- Perturbations and Mean Orbital Elements
- Hill's Equations
- Initial Conditions
- A Fuel Balancing Control Concept
- Formation Establishment and Maintenance
- High-Eccentricity Orbits
- Work in Progress
- Concluding Remarks

Global Positioning System (GPS) Constellation



No Inclination Difference

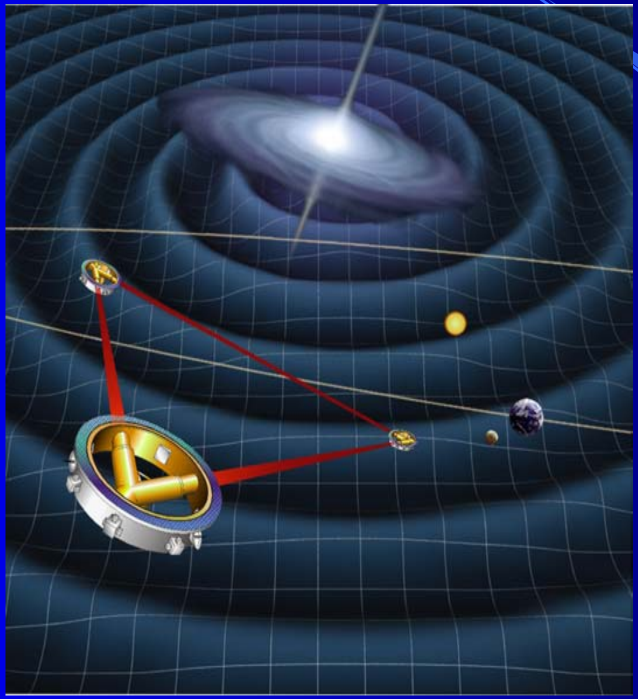
Formation Flying: Relative Orbits



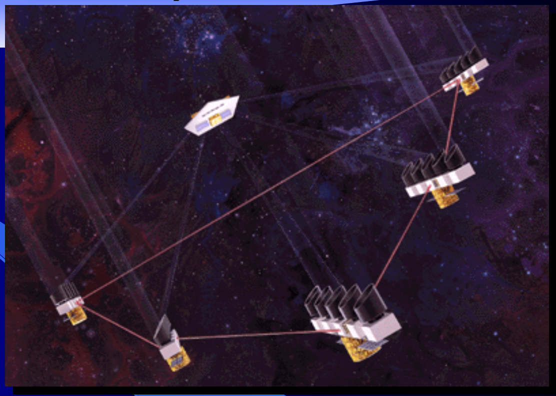
Distributed Space Systems- Enabling New Earth & Space Science (NASA)

Co-observation

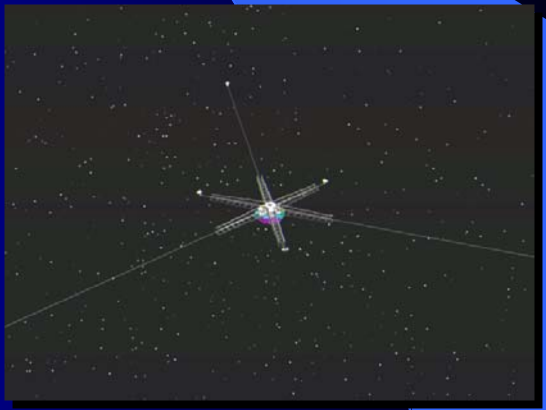
Multi-point observation



Large Interferometric Space Antennas



Interferometry



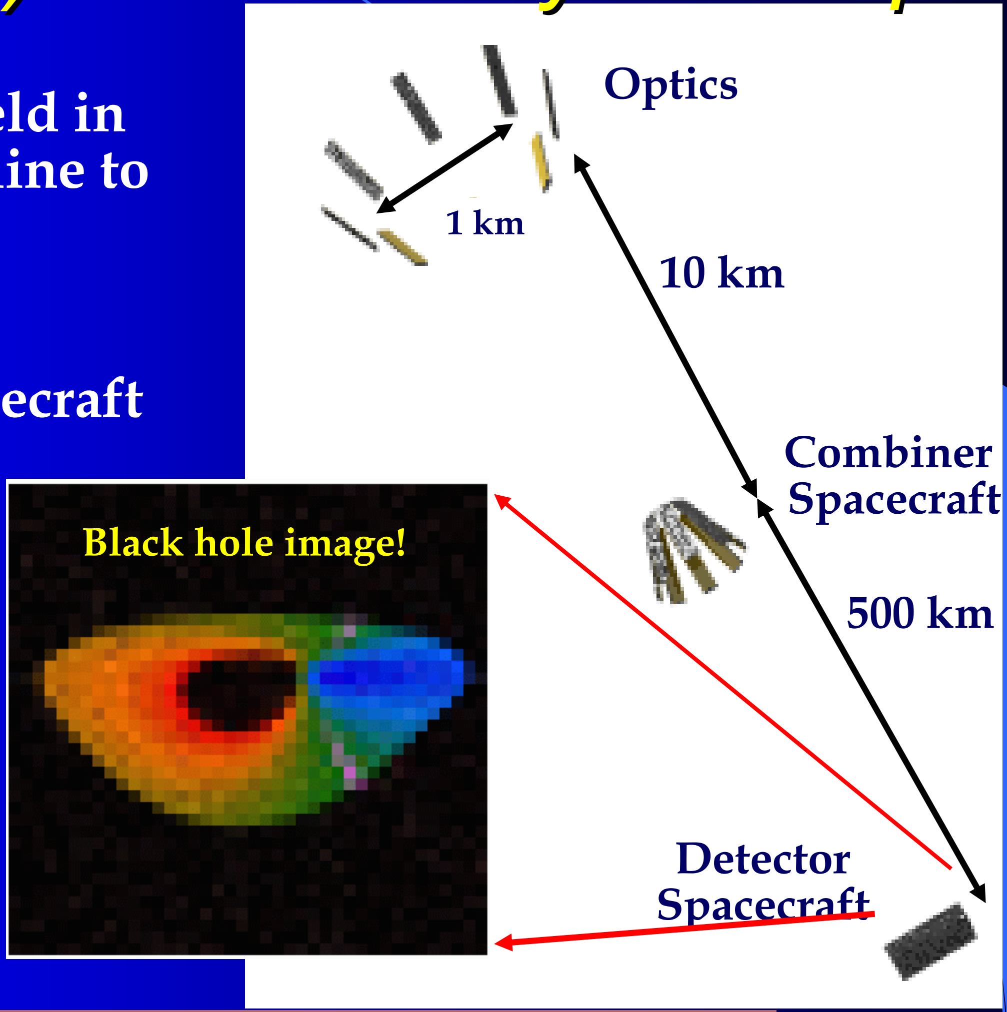
Tethered Interferometry

The Black Hole Imager: Micro Arcsecond X-ray Imaging Mission (MAXIM) Observatory Concept

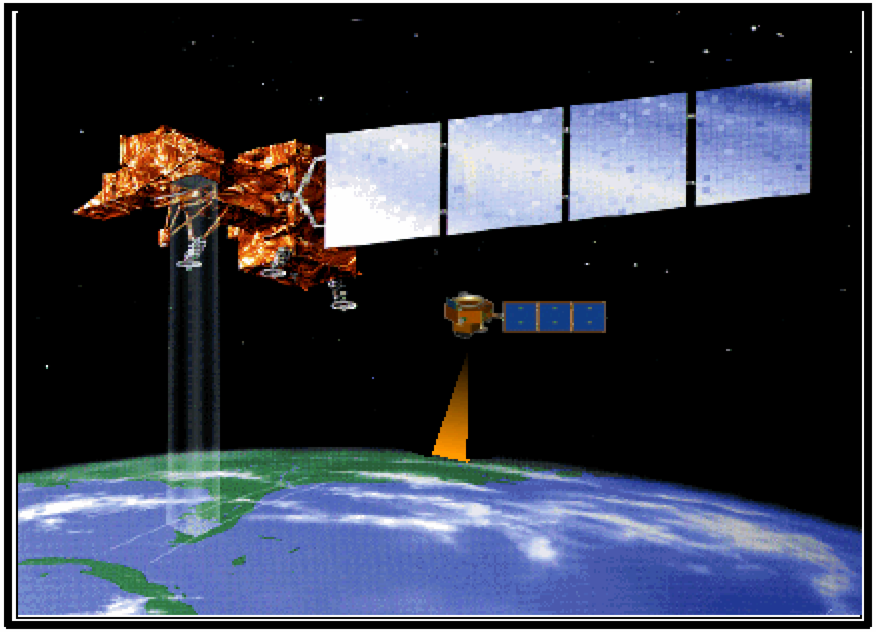
32 optics (300 × 10 cm) held in phase with 600 m baseline to give
0.3 micro arc-sec

34 Formation Flying Spacecraft

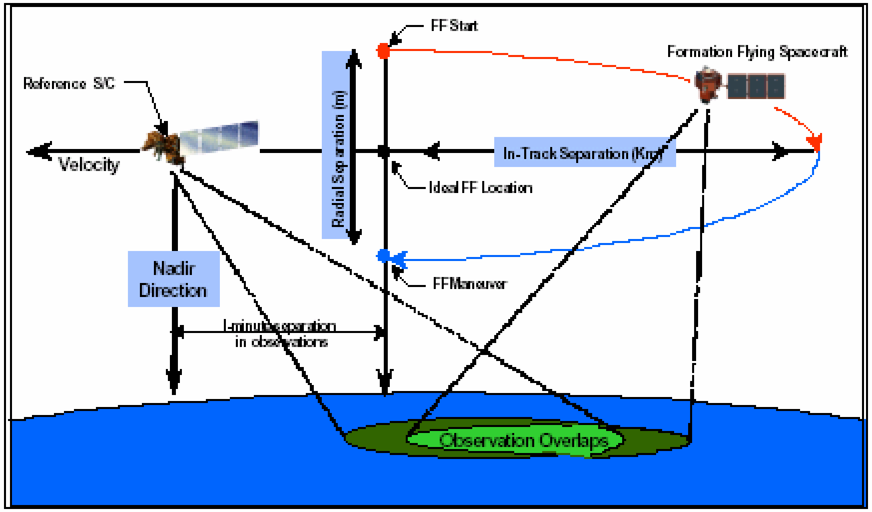
System is adjustable on orbit to achieve larger baselines



Landsat7/EO-1 Formation Flying



EO-1 Following Landsat-7 satellite in orbit



Orbit Mechanics of EO-1 Formation Flying

450 km in-track and 50m Radial Separation.

Differential Drag and Thrust Used for Formation Maintenance

Motivation for Research

- **Air Force: Sparse Aperture Radar.**
- **NASA and ESA:**

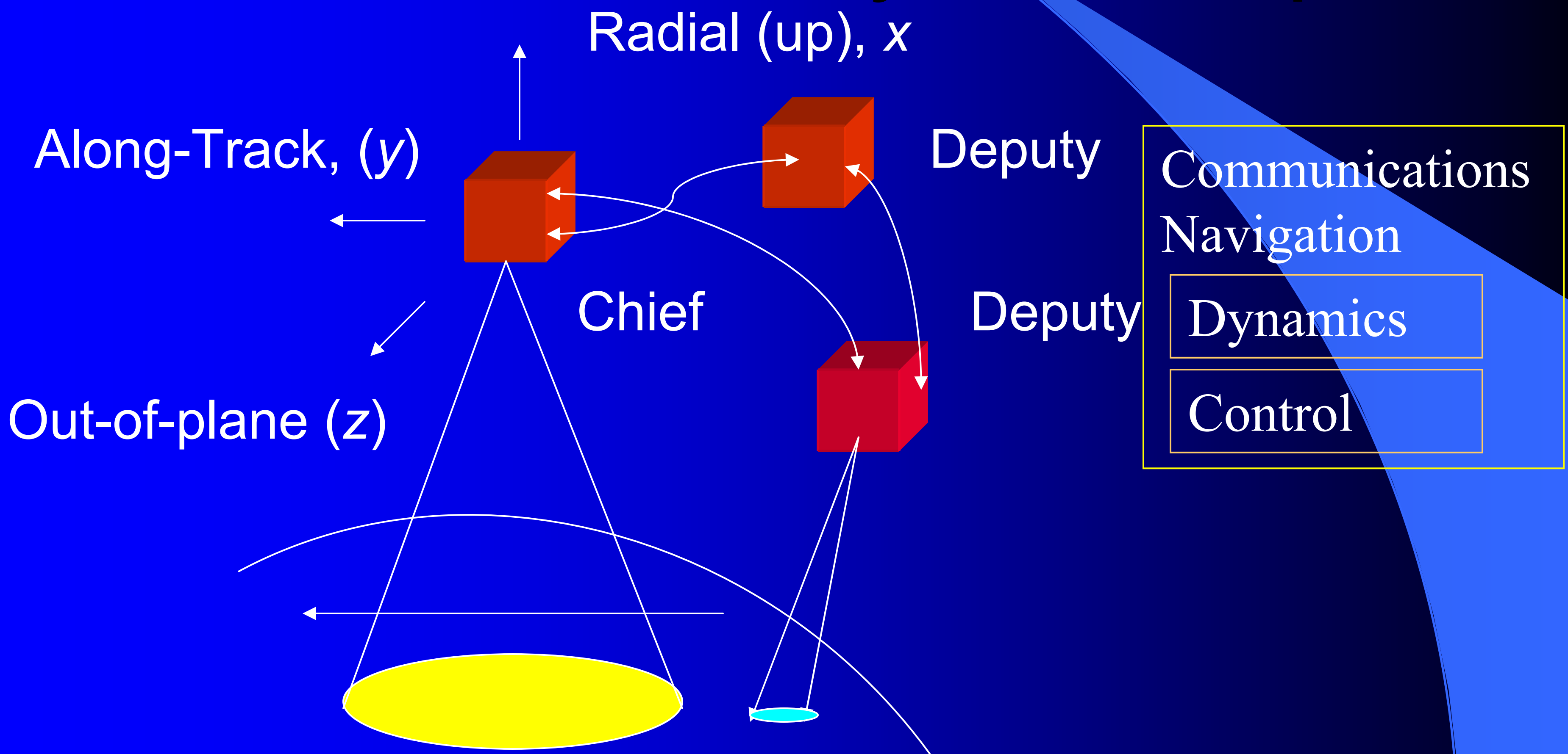
  Terrestrial Planet Finder (TPF)

  Stellar Imager (SI)

  LISA, MMS, Maxim
- **Swarms of small satellites flying in precise formations will cooperate to form distributed aperture systems.**
- **Determine Fuel efficient relative orbits. Do not fight Kepler!!!**
- **Effect of J_2?**
- **How to establish and reconfigure a formation?**
- **Balance the fuel consumption for each satellite and minimize the total fuel.**

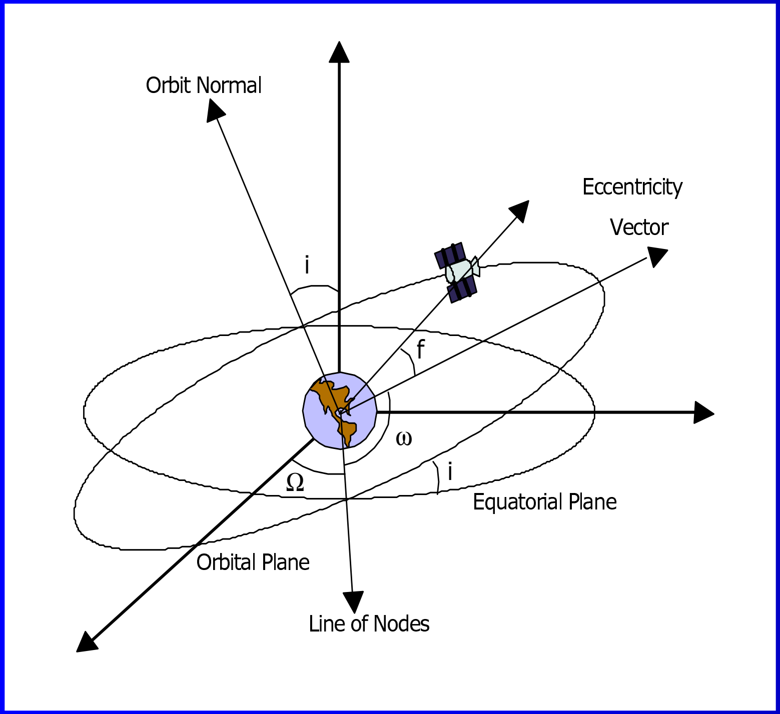
Introduction to Orbital Mechanics-1

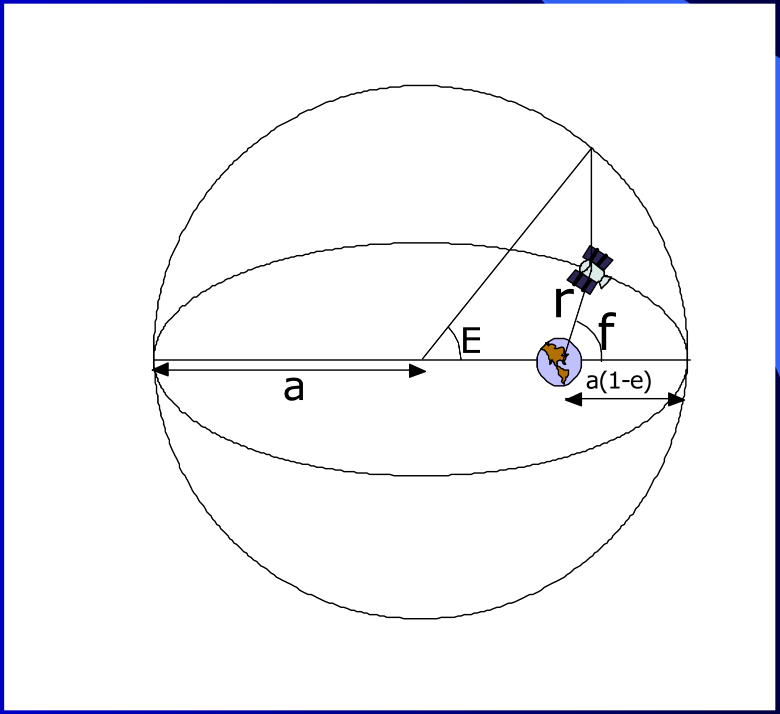
- **Formation Flying: Satellites close to each other but not necessarily in the same plane.**



Introduction to Orbital Mechanics-2

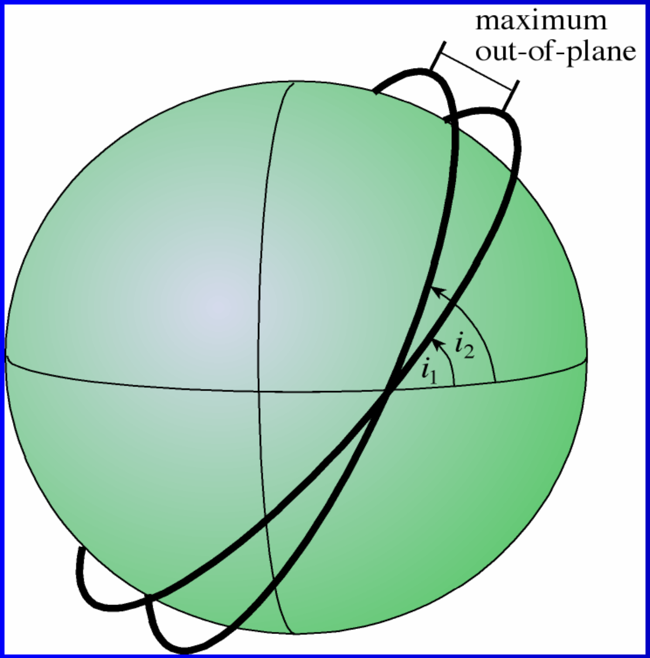
- **Orbital Elements: Five of the six elements remain constant for the 2-Body Problem.**
- **Variations exist in the definition of the elements.**
- **Mean anomaly:** $M : \dot{M} = n = \sqrt{\dfrac{\mu}{a^3}}$



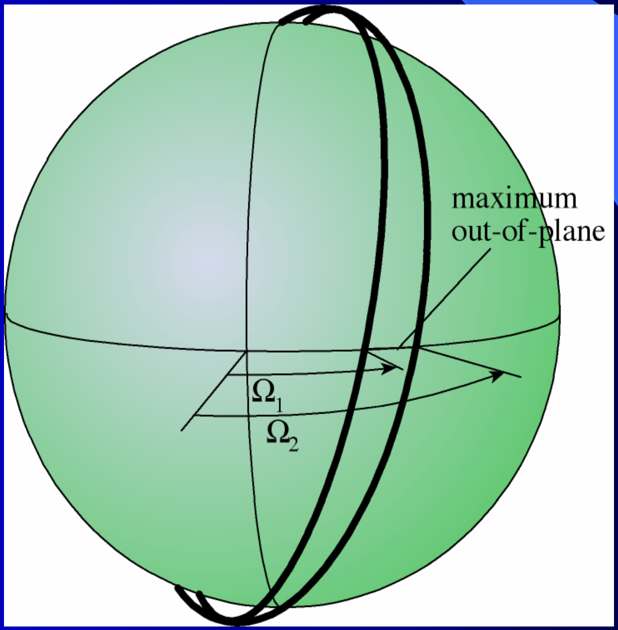


Orbital Mechanics-3

- **Ways to setup a formation:**
  - ***Inclination difference.***
  - ***Node difference.***
  - ***Combination of the two.***



Inclination Difference

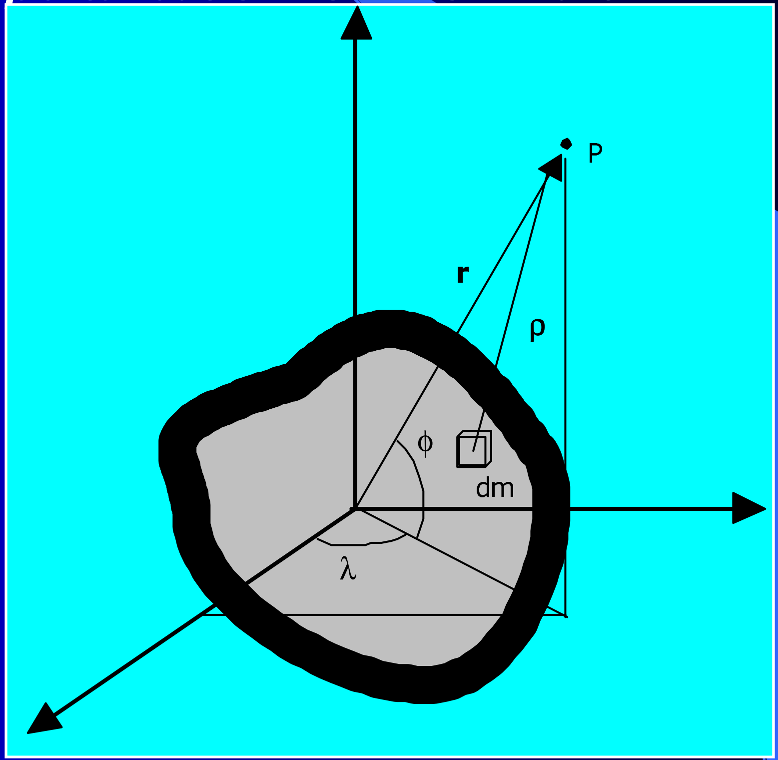


Node Difference

Orbital Mechanics-4

- **J_2 Perturbation:**
  - ***Gravitational Potential:*** $$\Phi(r,\phi) = -\frac{\mu}{r}\left[1 + \sum_{k=2}^{\infty} J_k \left(\frac{R_e}{r}\right)^k P_k(\sin\phi)\right]$$
  - ***J_2 is a source of a major perturbation on Low-Earth satellites .***
  - $$\Phi(r,\phi)\Big|_{J_2} = -\frac{\mu J_2 R_e^2}{2r^3}(3\sin\phi^2 - 1)$$ $$J_2 = 1.082629 \times 10^{-3}$$

Equatorial Bulge



Potential of an Aspherical body

Orbital Mechanics-5

- J_2 induces short and long periodic oscillations and secular Drifts in some of the orbital elements
- Secular Drift Rates

  - ***Node:*** $\dot{\Omega} = -1.5 J_2 \left(\frac{R_e}{p} \right)^2 n \cos i$
  - ***Perigee:*** $\dot{\omega} = 0.75 J_2 \left(\frac{R_e}{p} \right)^2 n \left(5 \cos^2 i - 1 \right)$
  - ***Mean anomaly:*** $\dot{M} = n + 0.75 J_2 \sqrt{1 - e^2} \left(\frac{R_e}{p} \right)^2 n \left(3 \cos^2 i - 1 \right)$

Drift rates depend on mean *a*, *e*, and *i*

Orbital Mechanics-6

- Analytical theories exist for obtaining Osculating elements from the Mean elements.
- Brouwer (1959)
- If two satellites are to stay close, their periods must be the same (2-Body).
- Under J_2 the drift rates must match.
- Requirements:

$$\dot{\Omega}_1 = \dot{\Omega}_2$$

$$\dot{\omega}_1 = \dot{\omega}_2$$

$$\boxed{\dot{M}_1 = \dot{M}_2}$$

Orbital Mechanics-7

- For small differences in a, e, and i

$$\begin{bmatrix} \dfrac{\partial \dot{\Omega}}{\partial a} & \dfrac{\partial \dot{\Omega}}{\partial e} & \dfrac{\partial \dot{\Omega}}{\partial i} \\ \dfrac{\partial \dot{\omega}}{\partial a} & \dfrac{\partial \dot{\omega}}{\partial e} & \dfrac{\partial \dot{\omega}}{\partial i} \\ \dfrac{\partial \dot{M}}{\partial a} & \dfrac{\partial \dot{M}}{\partial e} & \dfrac{\partial \dot{M}}{\partial i} \end{bmatrix} \begin{bmatrix} \delta a \\ \delta e \\ \delta i \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

- Except for trivial cases, all the three equations above cannot be satisfied with non-zero a, e, and i elemental differences.
- Need to relax one or more of the requirements.

Orbital Mechanics-8

- J_2-invariant Relative Orbits (Schaub and Alfriend, 2001).

$$\begin{bmatrix} \dfrac{\partial \dot{\Omega}}{\partial a} & \dfrac{\partial \dot{\Omega}}{\partial e} & \dfrac{\partial \dot{\Omega}}{\partial i} \\ \dfrac{\partial (\dot{\omega} + \dot{M})}{\partial a} & \dfrac{\partial (\dot{\omega} + \dot{M})}{\partial e} & \dfrac{\partial (\dot{\omega} + \dot{M})}{\partial i} \end{bmatrix} \begin{bmatrix} \delta a \\ \delta e \\ \delta i \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

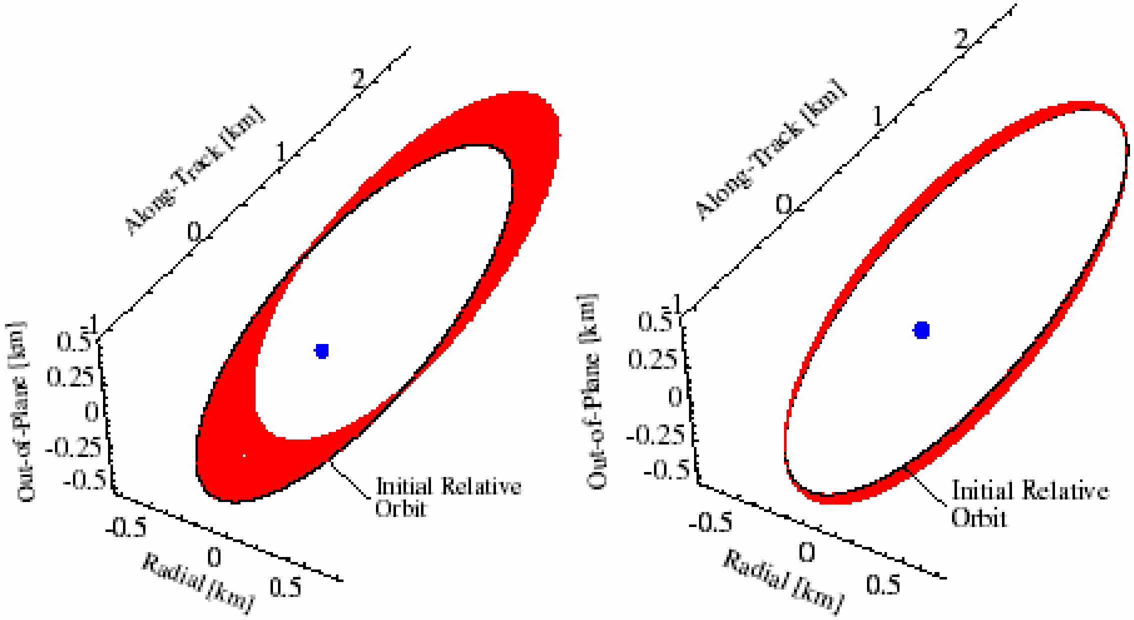
- This condition can sometimes lead to large relative orbits (For Polar Reference Orbits) or orbits that may not be desirable.

Orbital Mechanics-9

- J_2-invariant Relative Orbits (No Thrust Required)

TABLE I
MASTER SATELLITE ORBIT ELEMENTS.

Desired Average Orbit Elements	Value	Units
a	7153	km
e	0.05	
i	48 or 88	deg
h	0.0	deg
g	30.0	deg
l	0.0	deg



(a) Initial relative orbit setup in osculating elements

(b) Initial relative orbit setup in mean elements

Figure 2 Initial Relative Orbit Setup Comparison

Orbital Mechanics-10

- **Geometric Solution in terms of small orbital element differences**

$$y = a_c(\delta\theta + \delta\Omega\cos i_c); \quad \theta = \text{orbit latitude angle} = \omega + f$$

$$z = a_c(\delta i \sin\theta_c - \sin i_c \delta\Omega \cos\theta_c)$$

- **For small eccentricity**

$$\theta = \omega + M + 2e\sin M \text{ and}$$

$$\delta\theta = \delta\omega + \delta M + \boxed{2\delta e \sin M_c + 2e_c \cos M_c \delta M}$$

- **A condition for No Along-track Drift (Rate-Matching) is:**

$$\delta\omega + \delta M + \delta\Omega\cos i_c = 0$$

Remarks: Our Approach

- **The $\dot{\Omega}_0 = \dot{\Omega}_1$ and $\dot{\omega}_0 + \dot{M}_0 = \dot{\omega}_1 + \dot{M}_1$ constraints result in a large relative orbit for small eccentricity and high inclination of the Chief's orbit. (J_2-Invariant Orbits)**
- **Even if the inclination is small, the shape of the relative orbit may not be desirable.**
- **Use the no along-track drift condition (Rate-Matching) only.**
- **Setup the desired initial conditions and use as little fuel as possible to fight the perturbations.**

End of Phase-1

Hill-Clohessey-Wiltshire Equations-1

- Eccentric reference orbit relative motion dynamics (2-Body) :

$$\ddot{x} - 2\dot{\theta}\dot{y} - \ddot{\theta}y - \dot{\theta}^2 x = -\frac{\mu(r_c + x)}{[(r_c + x)^2 + y^2 + z^2]^{3/2}} + \frac{\mu}{r_c^2}$$

$$\ddot{y} + 2\dot{\theta}\dot{x} + \ddot{\theta}x - \dot{\theta}^2 y = -\frac{\mu y}{[(r_c + x)^2 + y^2 + z^2]^{3/2}}$$

$$\ddot{z} = -\frac{\mu z}{[(r_c + x)^2 + y^2 + z^2]^{3/2}}$$

- Assume zero-eccentricity and linearize the equations:

Hill-Clohessey-Wiltshire Equations-2

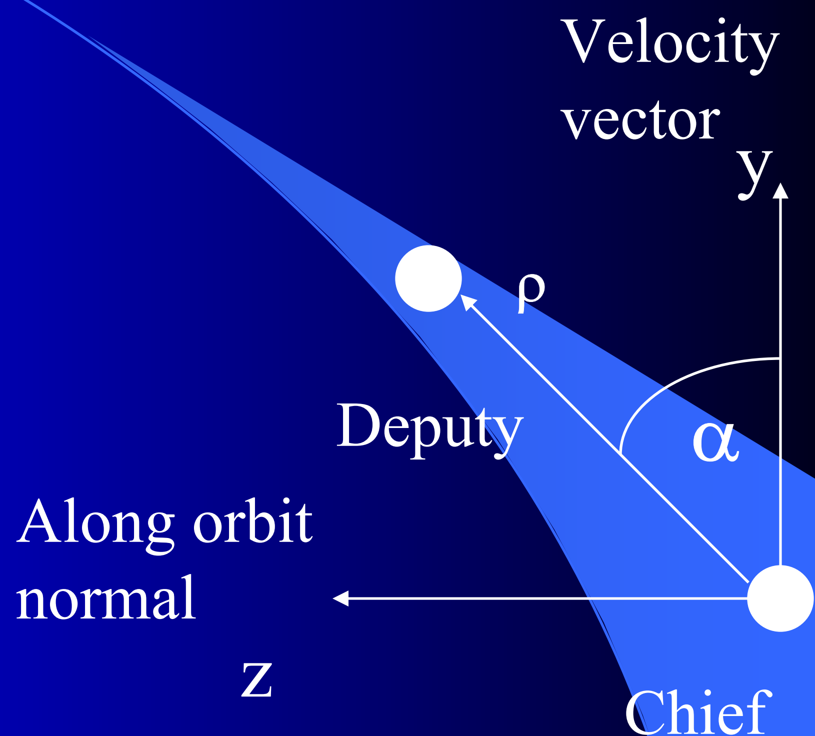
- HCW Equations:

$$\ddot{x} - 2n_c\dot{y} - 3n_c^{\ 2}x = 0$$

$$\ddot{y} + 2n_c\dot{x} = 0$$

$$\ddot{z} + n_c^{\ 2}z = 0$$

where $n_c = \sqrt{\dfrac{\mu}{a_c^3}}$.



- Bounded Along-Track Motion Condition

$$\dot{y} + 2n_c x = 0$$

Bounded HCW Solutions

$$x = \frac{c_1}{2}\sin(n_0 t + \alpha)$$

$$y = c_1\cos(n_0 t + \alpha) + c_3$$

$$z = c_2\sin(n_0 t + \alpha + \varphi)$$

- **Projected Circular Re. Orbit.**

$$c_1 = c_2 = \rho$$

$$c_3 = \varphi = 0$$

$$y^2 + z^2 = \rho^2$$

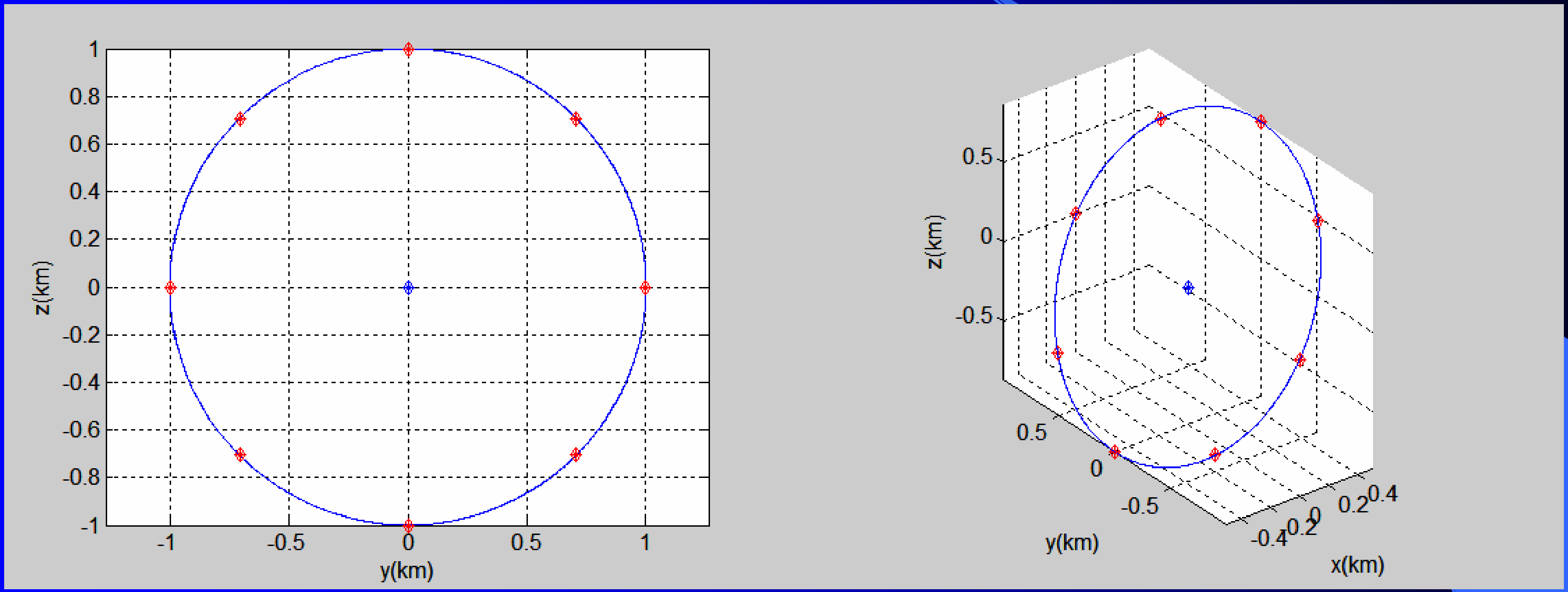
- **General Circular Re. Orbit.**

$$c_1 = \rho \qquad c_2 = \frac{\sqrt{3}}{2}\rho$$

$$c_3 = \varphi = 0$$

$$x^2 + y^2 + z^2 = \rho^2$$

PCO and GCO Relative Orbits



Projected Circular Orbit (PCO)

General Circular Orbit (GCO)

Initial Conditions in terms of Mean Element Differences : General Circular Relative Orbit.

$$\frac{\delta a}{a_c} = -\frac{J_2}{2}\left(\frac{R_e}{a_c}\right)^2 \frac{3\eta_c + 4}{\eta_c^4}\left[(1 - 3\cos^2 i_c)\frac{e_c \delta e}{\eta_c^2} + \sin 2i_c \delta i\right]$$

$$\eta_c = \sqrt{1 - e_c^2}$$

Semi-major axis Difference

$$\delta e = -0.5 c_1 \sin(\omega_c + \alpha)/a_c$$

Eccentricity Difference

$$\delta i = c_2 \cos(\alpha + \varphi)/a_c$$

Inclination Difference

$$\delta \Omega = -c_2 \frac{\sin(\alpha + \varphi)}{\sin i_c}/a_c$$

Node Difference

$$\delta \omega = ([c_2 \cot i_c \sin(\alpha + \varphi) - \frac{c_1}{2 e_c}\cos(\omega_c + \alpha)] + c_3)/a_c$$

Perigee Difference

$$\delta M = \frac{c_1}{2 e_c}\cos(\omega_c + \alpha)/a_c$$

Mean Anomaly Difference

Simulation Model

- Equations of motion for one satellite

$$\ddot{\mathbf{r}} = -\phi_r + \mathbf{u} \qquad \phi_r = \frac{\mu}{r^3}\mathbf{r} + \frac{J_2 \mu R_e^2}{2}\left[\frac{6z}{r^5}\hat{\mathbf{n}} + \left(\frac{3}{r^5} - \frac{15z^2}{r^7}\right)\mathbf{r}\right]$$

Inertial Relative Displacement

$$\delta\,\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_0$$

Inertial Relative Velocity

$$\delta\,\mathbf{v} = \mathbf{v}_1 - \mathbf{v}_0$$

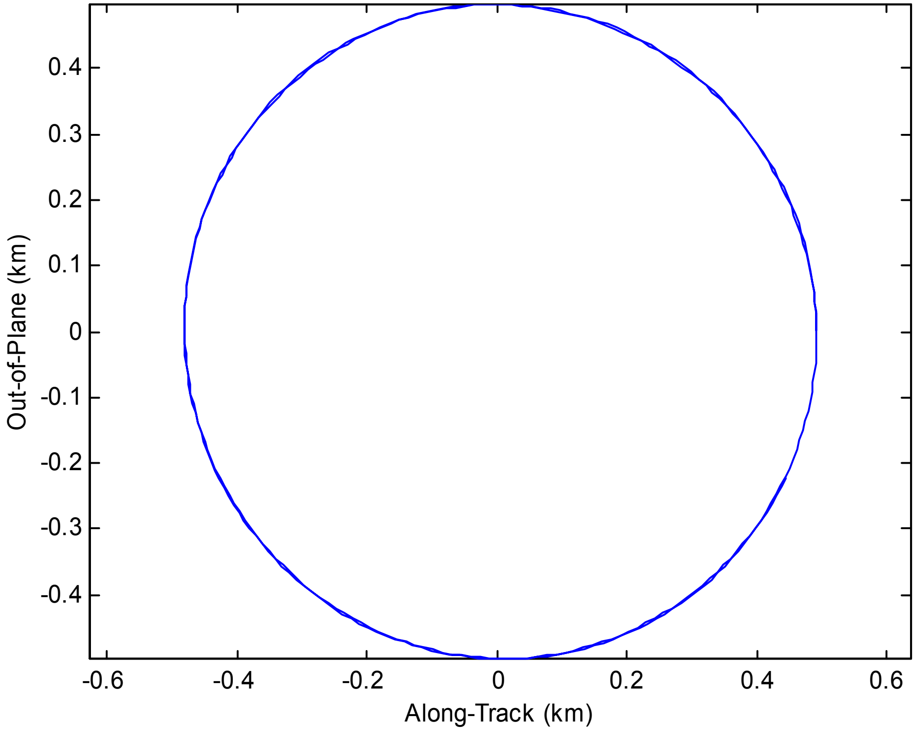
Rotating frame coordinates

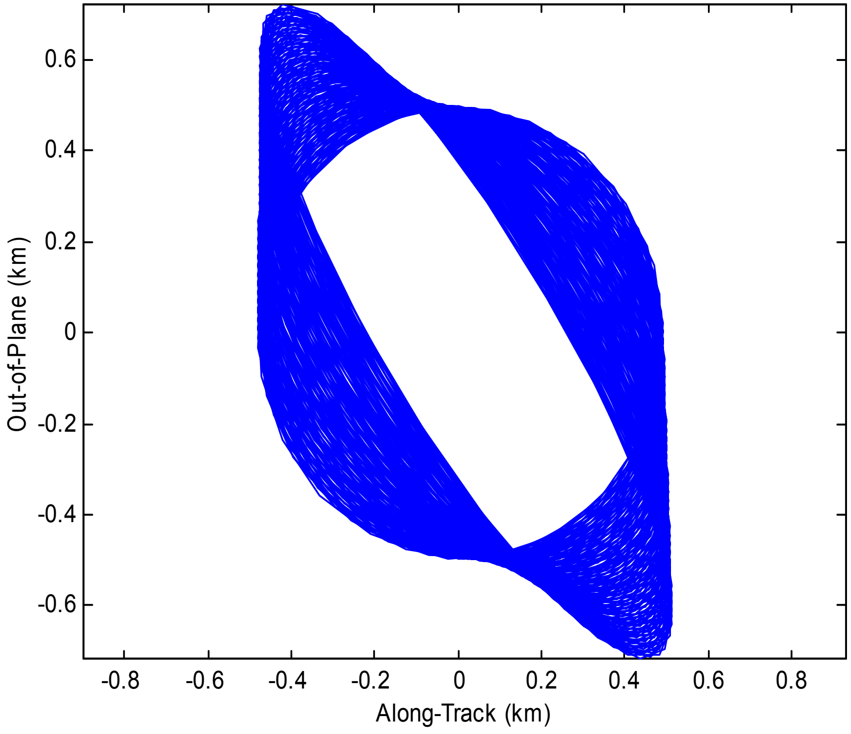
$$\delta\,x = \frac{\delta\,\mathbf{r}^T\mathbf{r}_0}{r_0} \qquad \delta\,y = \frac{\delta\,\mathbf{r}^T(\mathbf{H}_0 \times \mathbf{r}_0)}{|\mathbf{H}_0 \times \mathbf{r}_0|} \qquad \delta\,z = \frac{\delta\,\mathbf{r}^T\mathbf{H}_0}{H_0}$$

- Initial conditions: Convert Mean elements to Osculating elements and then find position and velocity.

Hill's Initial Conditions with Rate-Matching

- **Chief's orbit is eccentric: e=0.005**
- **Formation established using inclination difference only.** $\alpha_0 = 0^\circ$

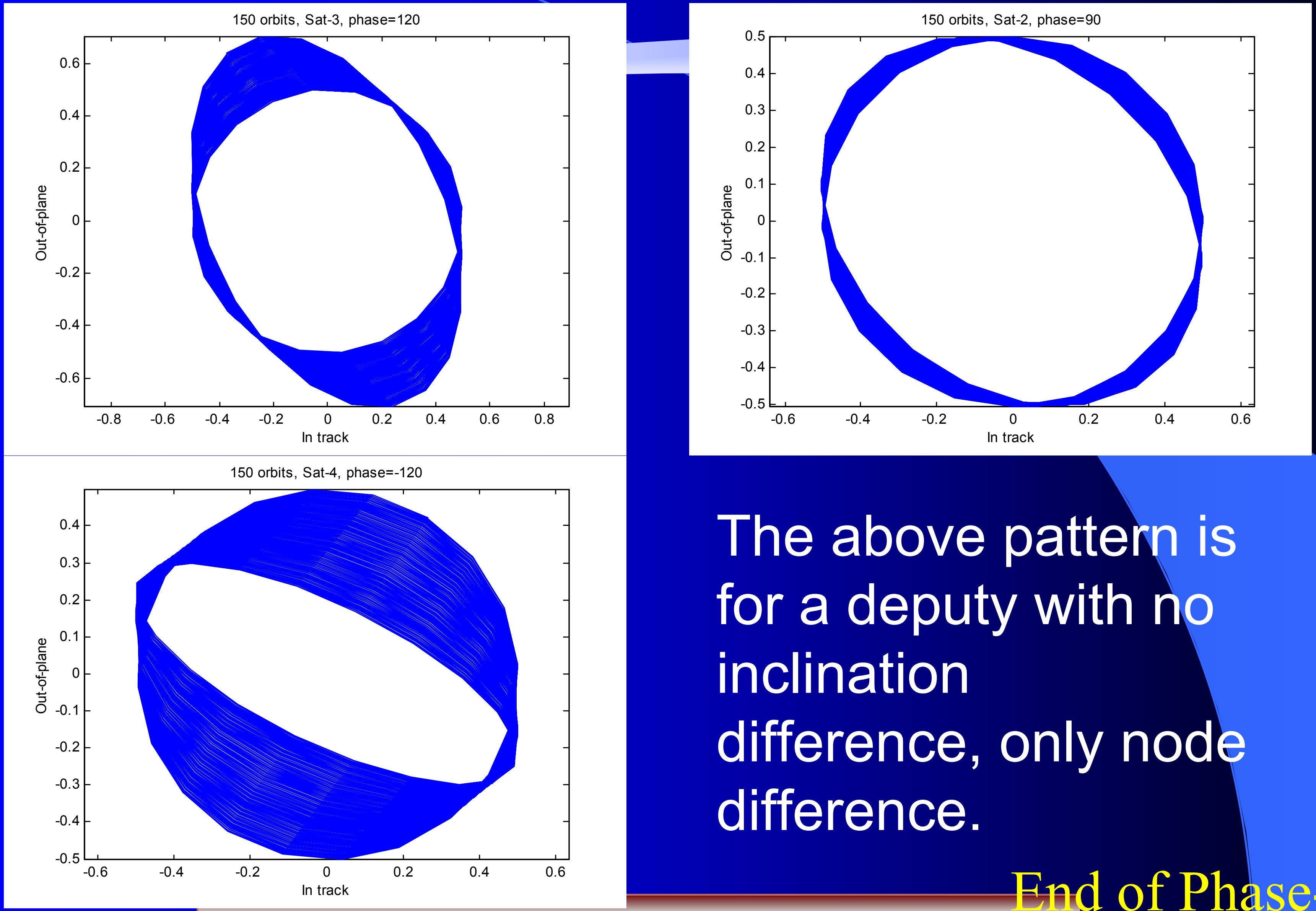




Relative Orbits in the y-z plane,
(2 orbits shown)

150 Relative Orbits

Drift Patterns for Various Initial Conditions



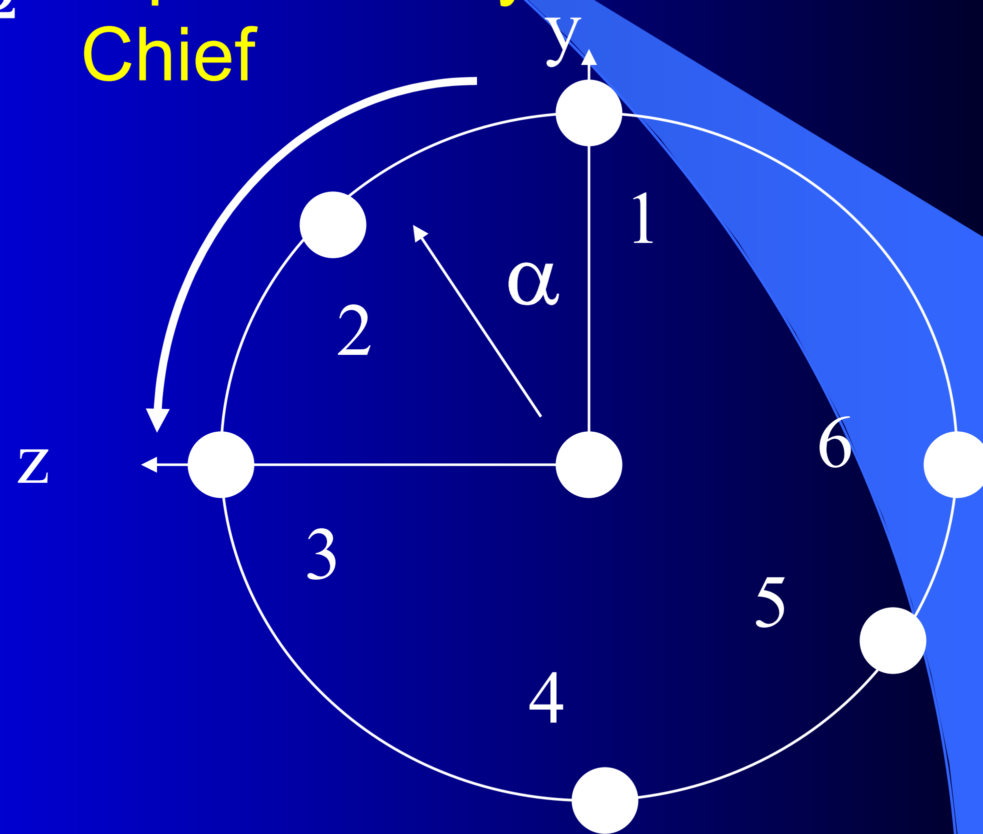
The above pattern is for a deputy with no inclination difference, only node difference.

End of Phase-2

Fuel Requirements for a Circular Projection Relative Orbit Formation

- Sat #1and 4 have max $|\delta i|$ and $\delta\Omega$ zero
- Sat #3 and 6 have max $|\delta\Omega|$ but zero δi
- 1 and 4 will spend max fuel; 3 and 6 will spend min fuel to fight J_2.

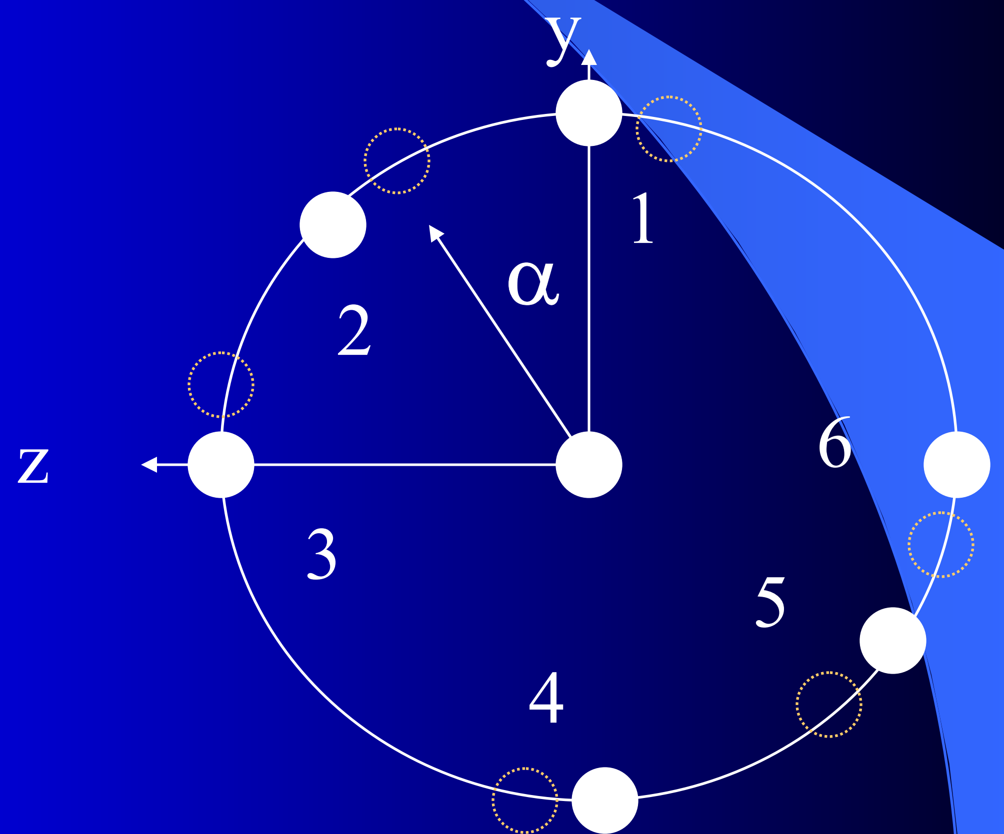
Snapshot when the chief is at the equator. Pattern repeats every orbit of the Chief



Fuel Balancing Control Concept

Snapshot when the chief is at the equator.

- Balance the fuel consumption over a certain period by rotating all the deputies by an additional rate $\dot{\alpha}$



Modified Hill's Equations to Account for J_2

$$\ddot{x} - 2\bar{n}_c \dot{y} - 3\bar{n}_c^2 x = u_x$$

$$\ddot{y} + 2\bar{n}_c \dot{y} = u_y$$

$$\ddot{z} + \bar{n}_c^2 z = u_z + 2A\bar{n}_c \cos\alpha_0 \sin\theta_c$$

$$A = \frac{3}{2}\rho\, J_2\, n_c \left(\frac{R_e}{a_c}\right)^2 \sin^2 i \qquad \bar{n}_c = \dot{\omega}_c + \dot{M}_c$$

Assume no in-track drift condition satisfied.

- Analytical solution

$$z = a_c(\delta i \sin\theta_c - \sin i_c \delta\Omega \cos\theta_c)$$

- The near-resonance in the z-axis is detuned by $\bar{n} \Rightarrow \bar{n} + \dot{\alpha}$

Balanced Formation Control Saves Fuel

- Ideal Trajectory

$$x_R = 0.5\rho\sin(\theta+\alpha)$$
$$y_R = \rho\cos(\theta+\alpha)$$
$$z_R = \rho\sin(\theta+\alpha)$$

- Optimize over time and an infinite number of satellites

- Ideal Control for perfect cancellation of the disturbance and for $\dot{\alpha}$

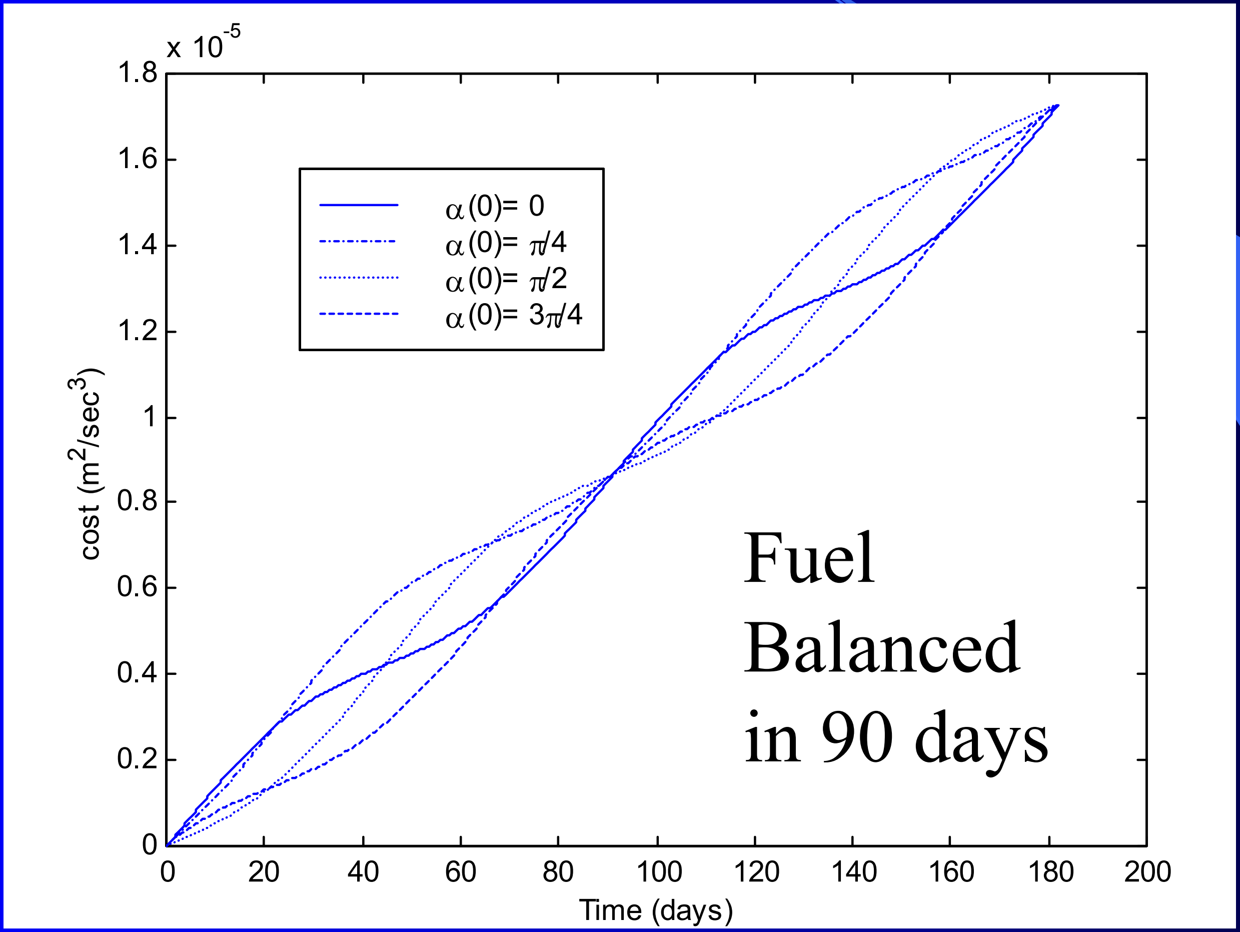
$$u_x = 2\bar{n}\dot{\alpha}x_R$$
$$u_y = -\bar{n}\dot{\alpha}y_R$$
$$u_z = -2\bar{n}\dot{\alpha}z_R - 2A\bar{n}\cos\alpha\sin\theta$$

$$J = \frac{1}{4\bar{n}\pi^2}\int_0^{2\pi}\int_0^{2\pi}\left(u_x^{\,2} + u_y^{\,2} + u_z^{\,2}\right)d\theta\, d\alpha_0$$

$$J_{opt} = \frac{2}{3}A^2\bar{n} \qquad \dot{\alpha}_{opt} = -\frac{A}{3\rho} \qquad J_{\dot{\alpha}=0} = A^2\bar{n}$$

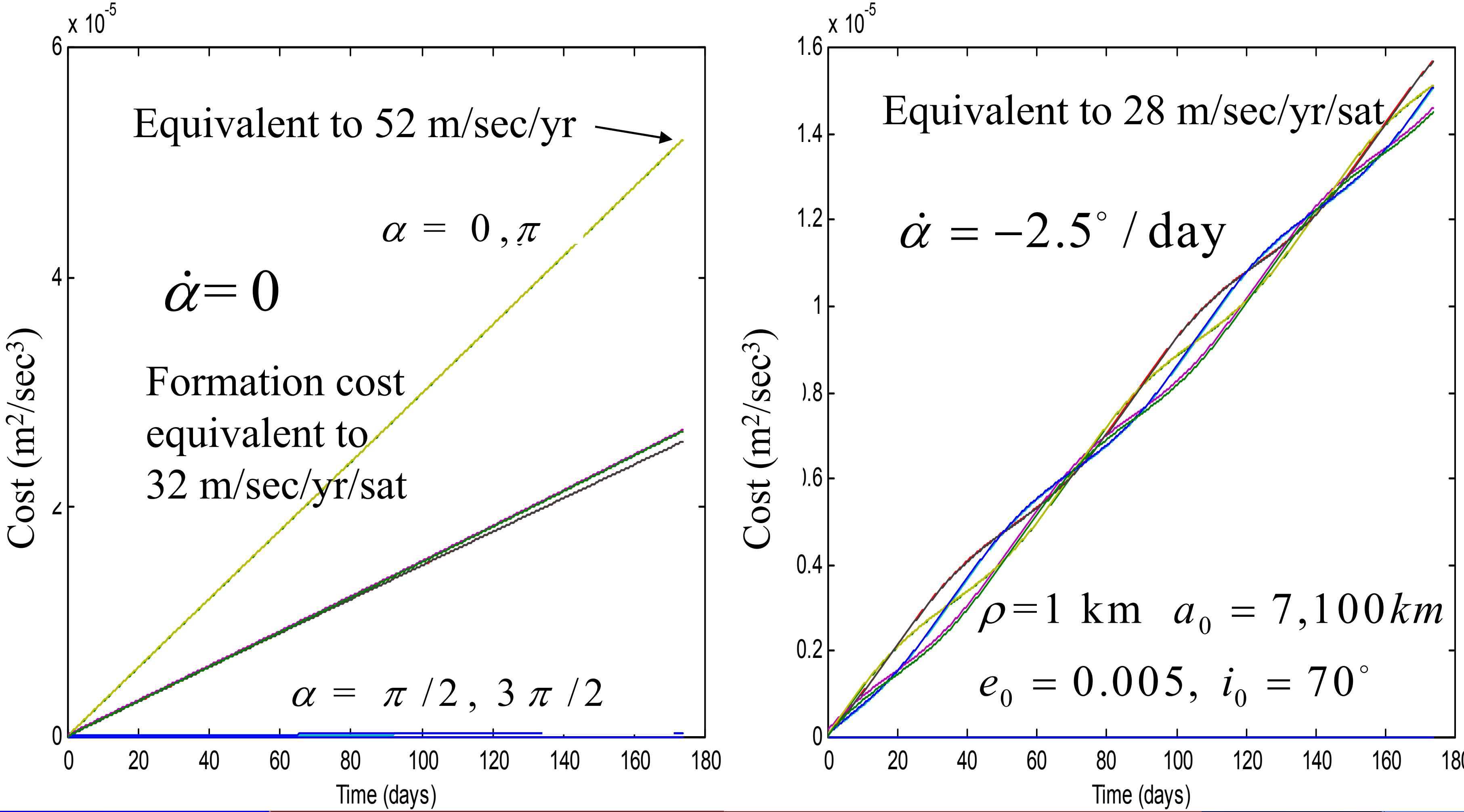
Analytical Results

Benefits of Rotation (Circular Projection Orbit)



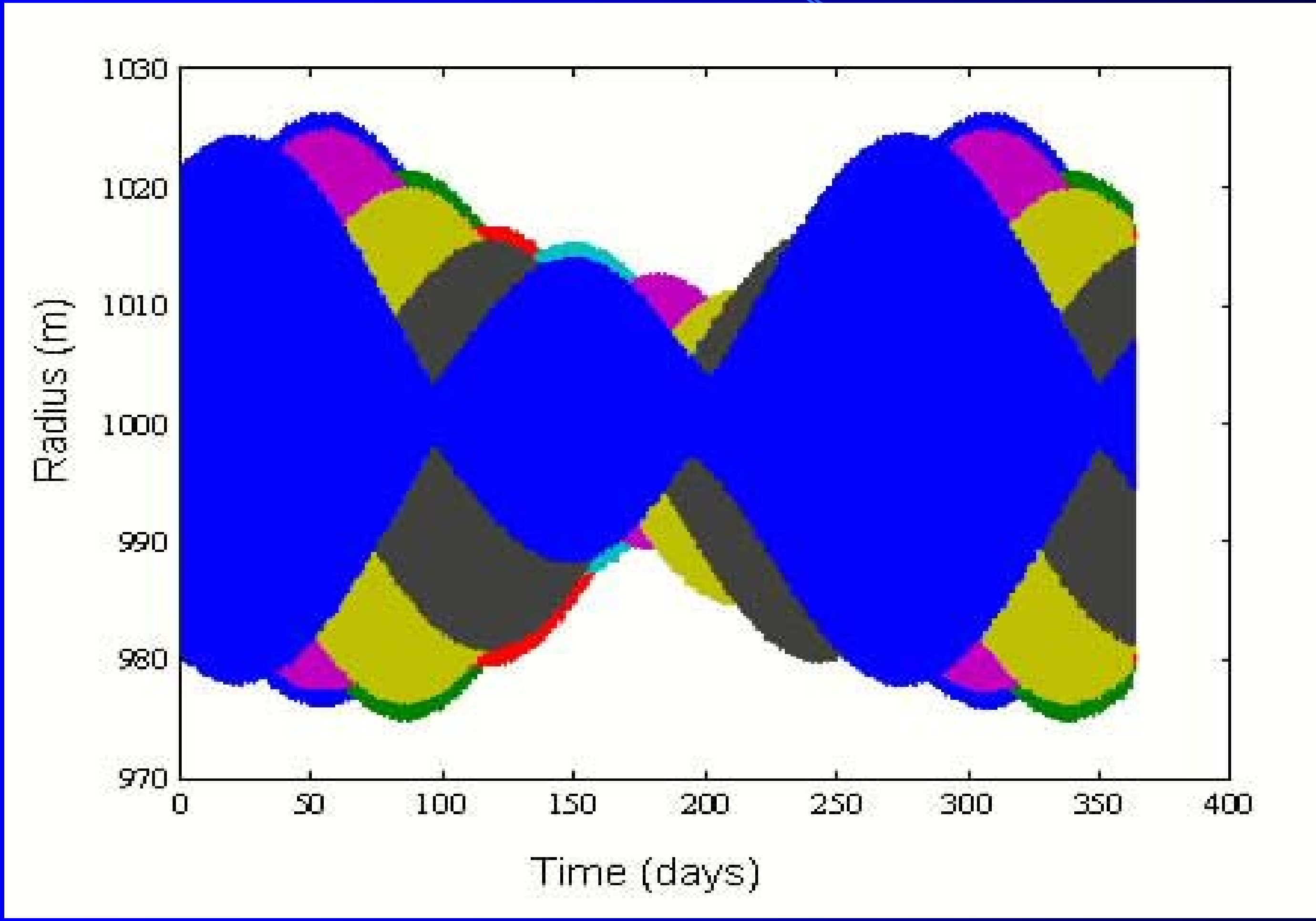
Nonlinear Simulation Results

Benefits of Rotation (Circular Projection Orbit)



Nonlinear Simulation Results

- **Orbit Radii over one year(8 Satellites)**



Disturbance Accommodation

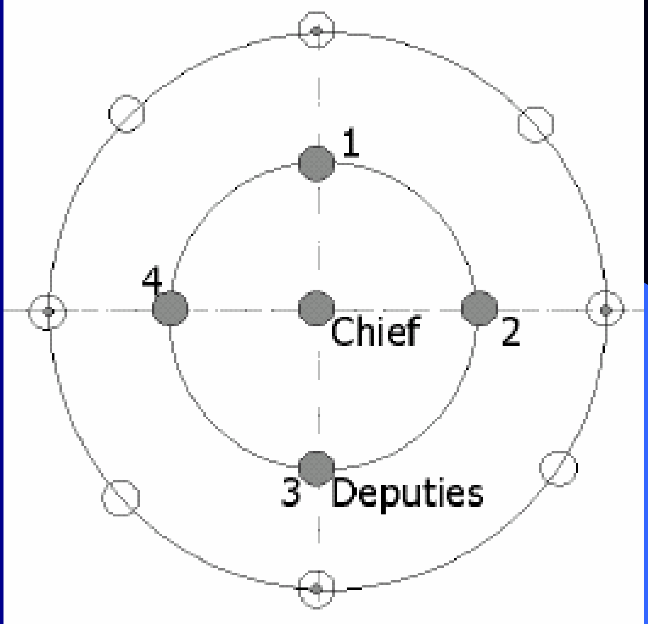
- Do not cancel J_2 and Eccentricity induced periodic disturbances above the orbit rate.
- Utilize Filter States
- No y-bias filter
- LQR Design
- Transform control to ECI and propagate orbits in ECI frame.
- The Chief is not controlled.

$$\dot{z}_{0x} = u_x$$
$$\ddot{z}_{2x} + (2\bar{n}_0)^2 z_{2x} = u_x$$
$$\ddot{z}_{3x} + (3\bar{n}_0)^2 z_{2x} = u_x$$
$$\ddot{z}_{2y} + (2\bar{n}_0)^2 z_{2y} = u_y$$
$$\ddot{z}_{3y} + (3\bar{n}_0)^2 z_{3y} = u_y$$
$$\dot{z}_{0z} = u_z$$
$$\ddot{z}_{2z} + (2\bar{n}_0)^2 z_{2z} = u_z$$
$$\ddot{z}_{3z} + (3\bar{n}_0)^2 z_{3z} = u_z$$

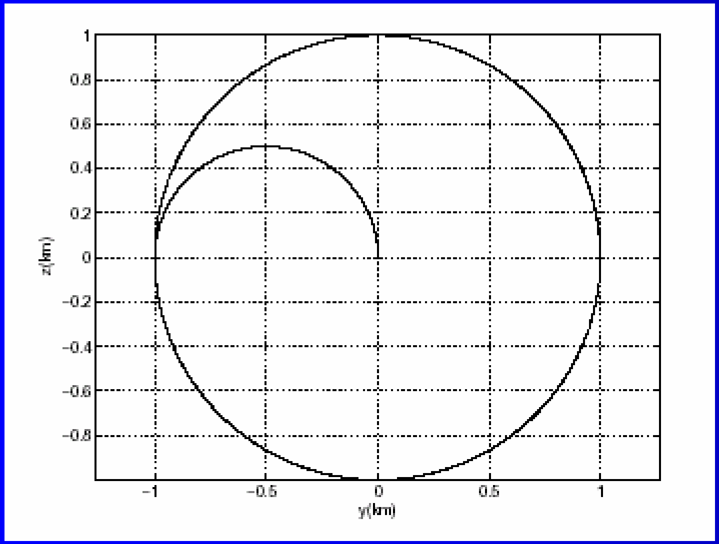
End of Phase-3

Formation Establishment and Reconfiguration

- Changing the Size and Shape of the Relative Orbit.
- Can be Achieved by a 2-Impulse Transfer.
- Analytical solutions match numerically optimized Results.
- Gauss' Equations Utilized for Determining Impulse magnitudes, directions, and application times.
- Assumption: The out-of-plane cost dominates the in-plane cost. Node change best done at the poles and inclination at the equator crossings.

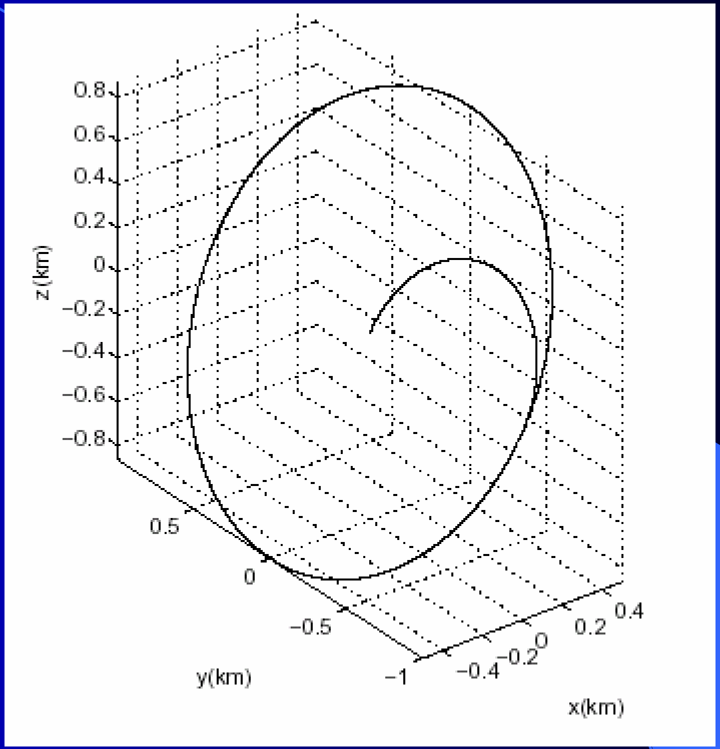


Formation Establishment



1 km PCO

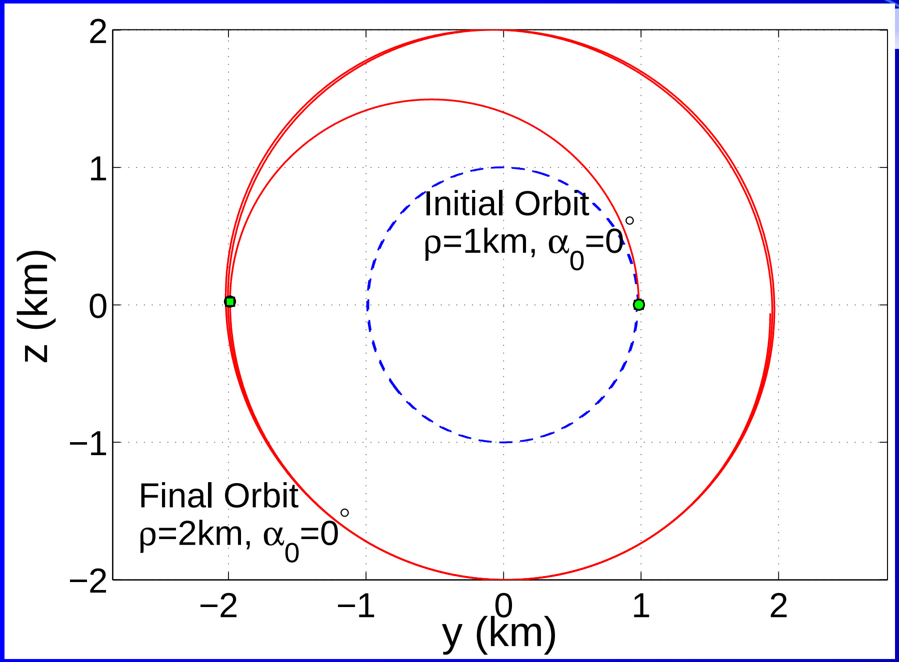
Established with $\alpha_0 = 45^\circ$

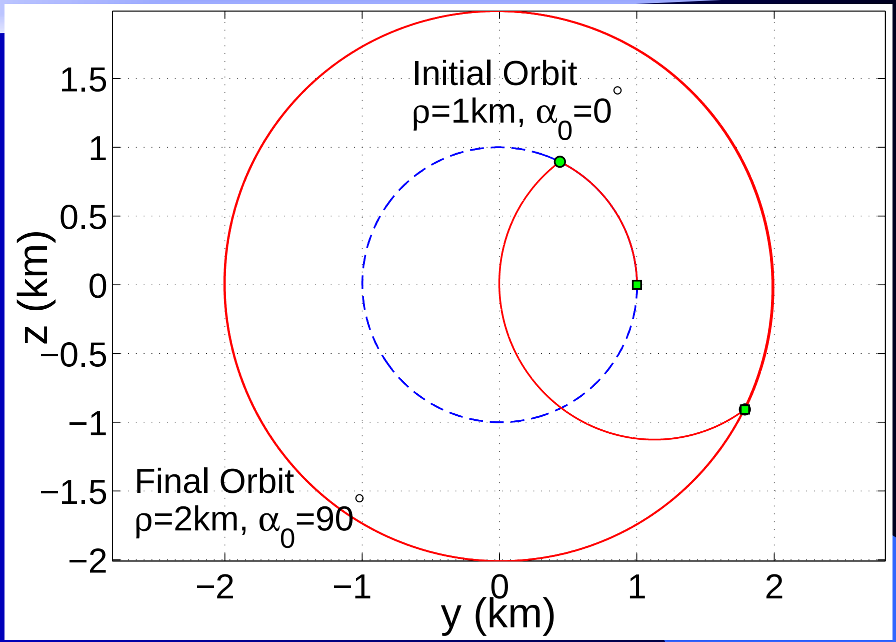


1 km GCO

Established with $\alpha_0 = 90^\circ$

Formation Reconfiguration



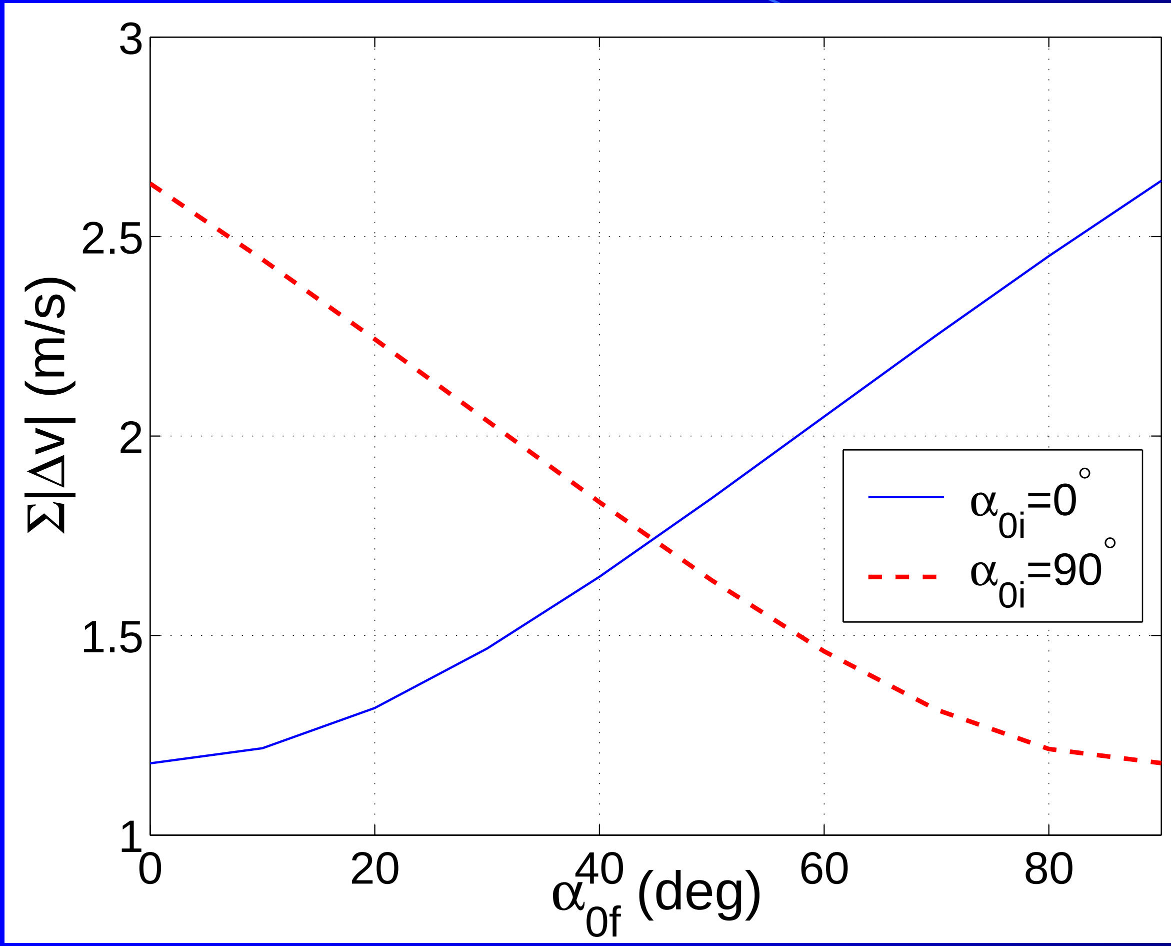


1 km, $\alpha_0 = 0^\circ$ PCO to 2 km, $\alpha_0 = 0^\circ$ PCO

1 km, $\alpha_0 = 0^\circ$ PCO to 2 km, $\alpha_0 = 90^\circ$ PCO

Chief is at the Asc. Node at the Beginning.

Reconfiguration Cost



Cost vs. Final Phase Angle

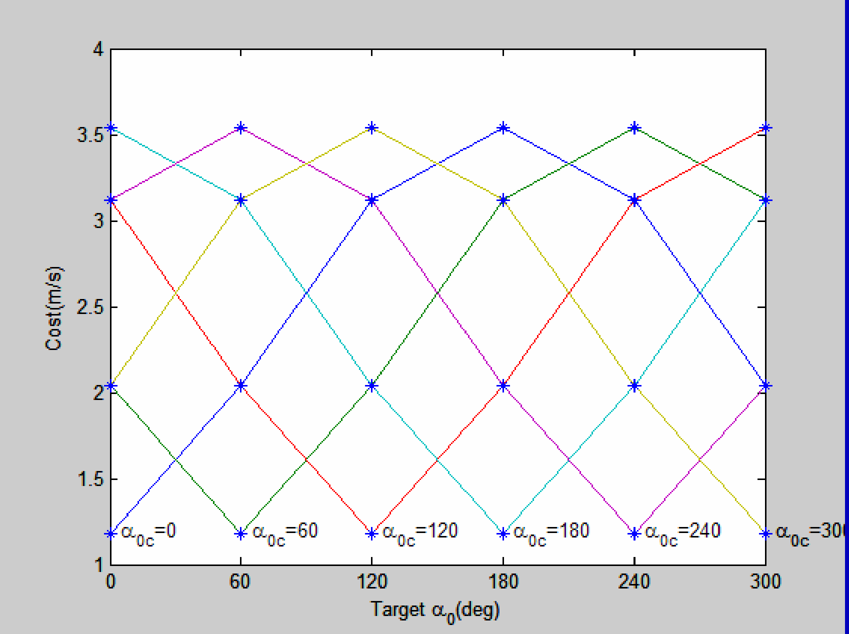
This plot helps in solving the slot assignment problem. The initial and final phase angles should be the same for fuel optimality for any initial phase angle.

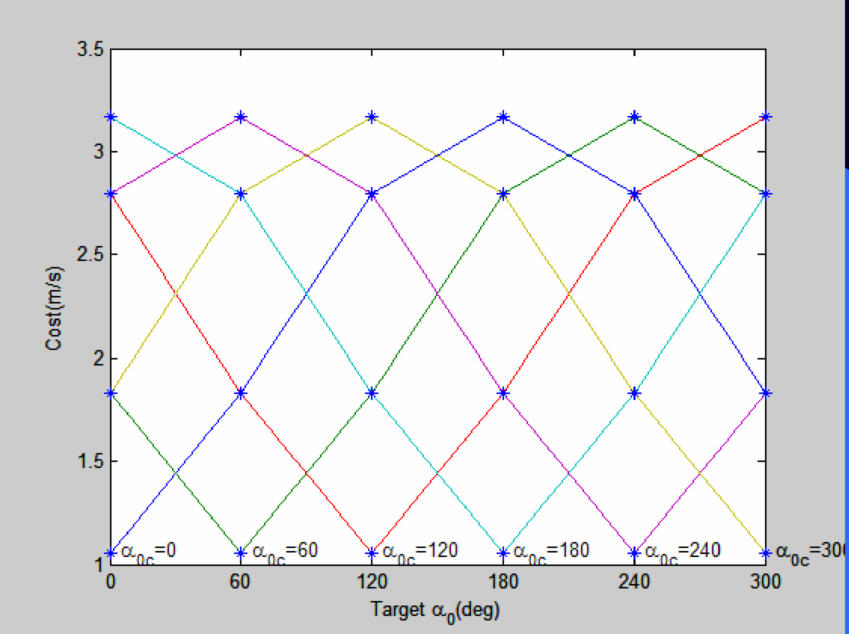
Optimal Assignment

Objectives:

(i) Minimize Overall Fuel Consumption

(ii) Homogenize Individual Fuel Consumption





PCO $\rho_i = 1$ to $\rho_f = 2$

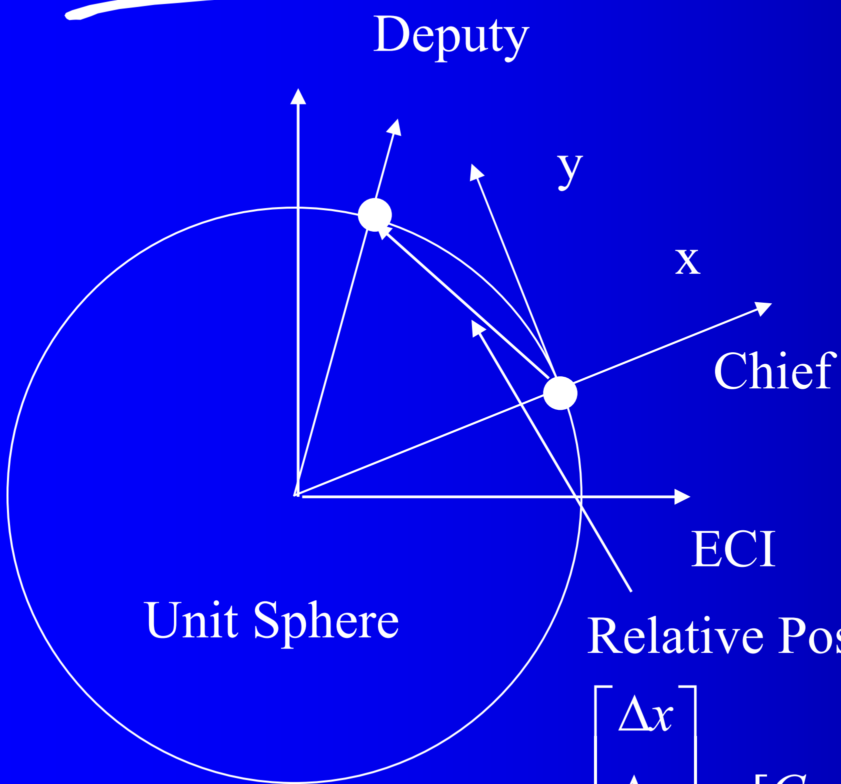
GCO $\rho_i = 1$ to $\rho_f = 2$

End of Phase-4

Relative Motion on a Unit Sphere

C_C : Attitude Matrix of the Chief

C_D : Attitude Matrix of the Deputy



Relative Position Vector

$$\begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} = [C_C C_D{}^T - I] \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Relative Motion Solution on the Unit Sphere

$$\Delta x = -1 + c^2(i_0/2)c^2(i_1/2)c(\Delta\theta + \Delta\Omega) + s^2(i_0/2)s^2(i_1/2)c(\Delta\theta - \Delta\Omega)$$
$$+ s^2(i_0/2)c^2(i_1/2)c(2\theta_0 + \Delta\theta + \Delta\Omega) + c^2(i_0/2)s^2(i_1/2)c(2\theta_0 + \Delta\theta - \Delta\Omega)$$
$$+ 1/2 s(i_0)s(i_1)[c(\Delta\theta) - c(2\theta_0 + \Delta\theta)]$$

$$\Delta y = c^2(i_0/2)c^2(i_1/2)s(\Delta\theta + \Delta\Omega) + s^2(i_0/2)s^2(i_1/2)s(\Delta\theta - \Delta\Omega)$$
$$- s^2(i_0/2)c^2(i_1/2)s(2\theta_0 + \Delta\theta + \Delta\Omega) - c^2(i_0/2)s^2(i_1/2)s(2\theta_0 + \Delta\theta - \Delta\Omega)$$
$$+ 1/2 s(i_0)s(i_1)[s(\Delta\theta) + s(2\theta_0 + \Delta\theta)]$$

$$\Delta z = -s(i_0)s(\Delta\Omega)c(\theta_1) - [s(i_0)c(i_1)c(\Delta\Omega) - c(i_0)s(i_1)]s(\theta_1)$$

- Valid for Large Angles

Analytical Solution using Mean Orbital Elements-1

- Mean rates are constant.

$$\theta_j = \omega_j + M_j + 2e_j \sin(M_j) + 5/4\, e_j^2 \sin(2M_j) + \ldots, \quad j = 0,1$$

$$\Omega_j = \Omega_j(0) + \dot{\Omega}_j t \qquad \dot{\Omega}_j = -1.5 J_2 (R_e / p_j)^2 n_j \cos(i_j)$$

$$\omega_j = \omega_j(0) + \dot{\omega}_j t \qquad \dot{\omega}_j = 0.75 J_2 (R_e / p_j)^2 n_j \left(5\cos^2 i_j - 1\right)$$

$$M_j = M_j(0) + \dot{M}_j t \qquad p_j = a_j(1 - e_j^2)$$

$$\dot{M}_j = n_j \left[1 + 0.75 J_2 \sqrt{1 - e_j^2}\,(R_e / p_j)^2 \left(3\cos^2 i_j - 1\right)\right]$$

Analytical Solution using Mean Orbital Elements-2

- Actual Relative Motion.

$$\delta x = r_1 (1 + \Delta x) - r_0$$

$$\delta y = \Delta y \ r_1$$

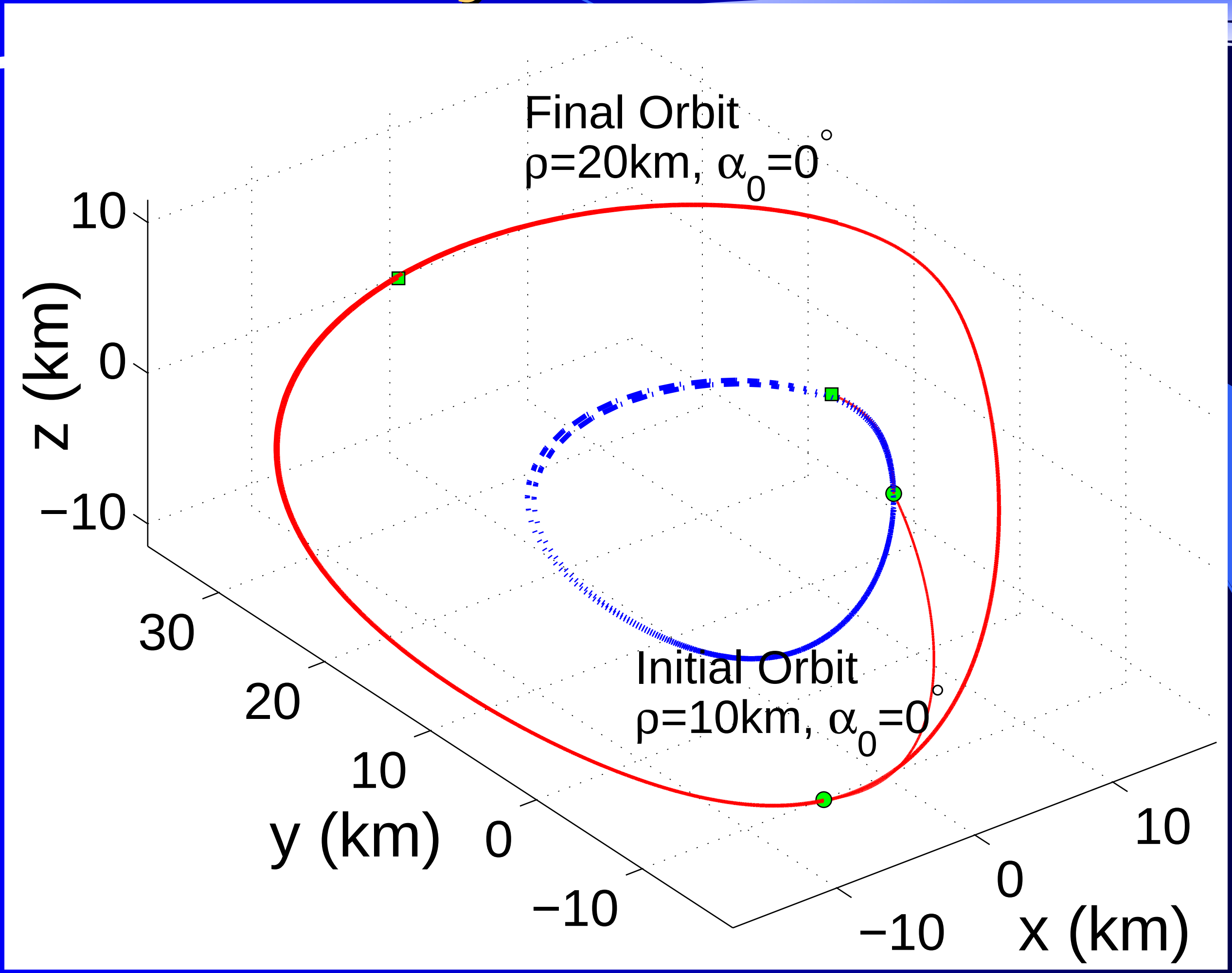
$$\delta z = \Delta z \ r_1$$

$$r_j = a_j[1 - e_j \cos(M_j) - 1/2 e_j{}^2 (\cos(2M_j) - 1) + ...], \ j = 0,1$$

High Eccentricity Reference Orbits

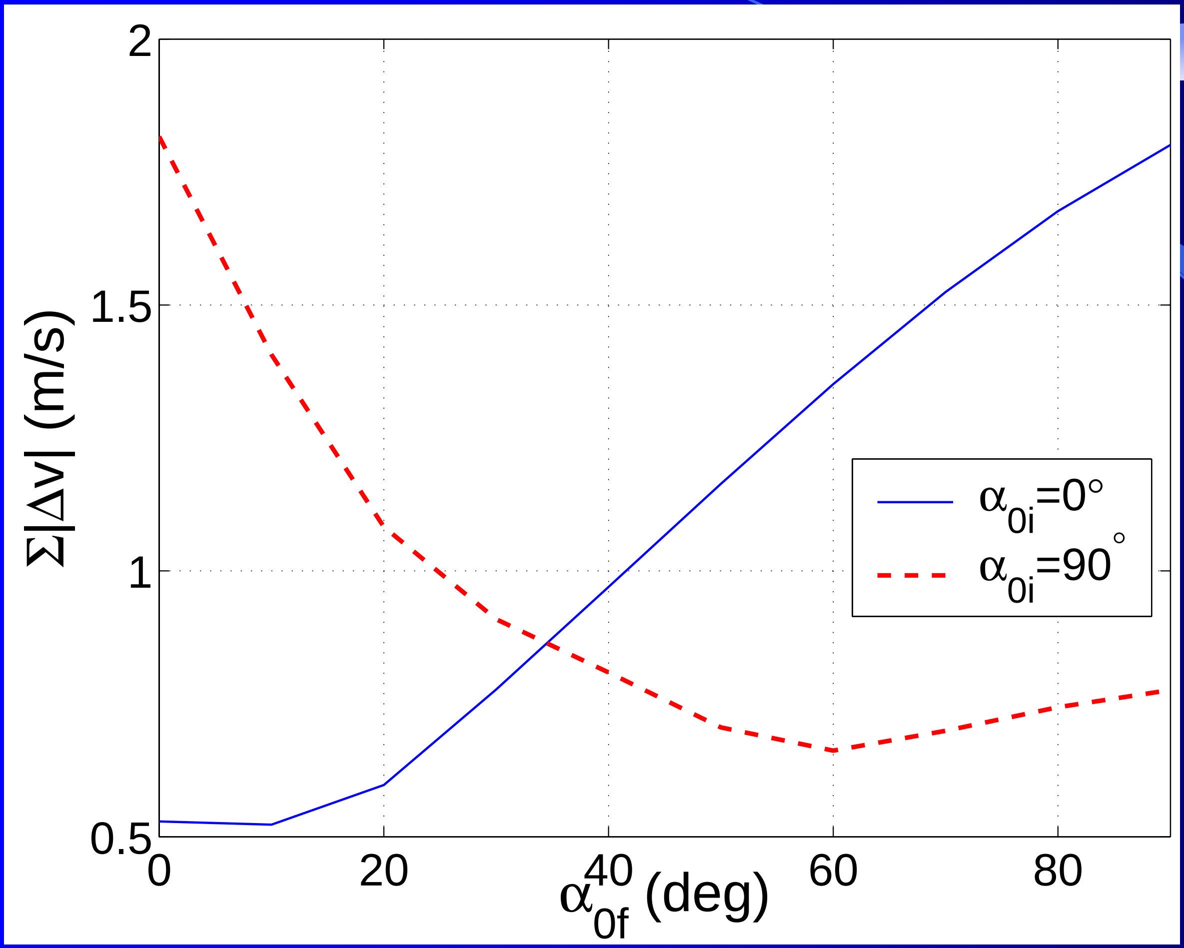
- Eccentricity expansions do not converge for high e.
- Use true anomaly as the independent variable and not time.
- Need to solve Kepler's equation for the Deputy at each data output point.

Formation Reconfiguration for High-Eccentricity Reference Orbits



August 19, 2003

High Eccentricity Reconfiguration Cost



Impulses are applied close to the apogee. No symmetry is observed with respect to phase angle.

Cost vs. Final Phase Angle

Research in Progress

- Higher order nonlinear theory and period matching conditions for large relative orbits.
- Continuous control Reconfiguration (Lyapunov Functions).
- Nonsingular Elements (To handle very small eccentricity)
- Earth-moon and sun-Earth Libration point Formation Flying.

Concluding Remarks

- Discussed Issues of Near-Earth Formation Flying and methods for formation design and maintenance.
- Spacecraft that have similar Ballistic coefficients will not see differential drag perturbations.
- Differential drag is important for dissimilar spacecraft (ISS and Inspection Vehicle).
- Design of Near-Earth Formations in high-eccentricity orbits pose many analytical challenges.
- Thanks for the opportunity and hope you enjoyed your lunch!!

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