Earth Risk Corridor Computations for 2011 AG$_5$ on 5 February 2040

Introduction

The near-Earth object (NEO) designated 2011 AG$_5$ is currently associated with a relatively high linearized Earth impact probability of 0.001557 (odds of 1-in-642.3) during its predicted 5 February 2040 close approach to Earth. This probability appears in Table 1 (reference the **`P_i/p`** column), together with data pertaining to other near-term 2011 AG$_5$ planetary approaches closer than 0.1 AU (15 million km). These predictions are obtained from JPL's *Horizons* on-line solar system data and ephemeris computation service[*] using the current JPL#45 orbit solution for 2011 AG$_5$. Note that coordinate time (CT) is a uniform time scale void of leap seconds and used as the fundamental ephemeris argument by *Horizons*. To a precision of ±0.002 s, CT is related to atomic time (TAI) by CT = TAI + 32.184 s.

Table 1. Planetary close approaches into the near future are predicted and displayed by the *Horizons* telnet interface for 2011 AG$_5$ beginning with the Earth approach during which it was discovered.

```
                                  AU        AU        AU       km/s    min
      Date (CT)        Body    CA Dist  MinDist   MaxDist    Vrel  TCA3Sg  Nsigs   P_i/p
   -----------------   -----   -------  -------   -------   ------ ------ ------ -------
   2011 Feb 26.64066   Earth   .095668  .095665   .095671    7.516   0.25 53669. .000000
   2016 Sep 02.27657   Mars    .067195  .067184   .067201   11.606  18.59 9.05E5 .000000
   2023 Feb 03.34331   Earth   .012380  .010517   .014232    9.925 286.37 4017.3 .000000
   2040 Feb 05.73432   Earth   .004544  .000012   .127709    9.473 16066. .14880 .001557
```

The **`MinDist`** and **`MaxDist`** columns in Table 1 encompass three standard deviations (3σ) in linearized covariance mapping about the "nominal" (undispersed or best-guess) closest approach distance in the **`CA Dist`** column. Variations among the three closest approach distance columns grow dramatically after the Mars approach in 2016 and the intermediate Earth approach in 2023.

Typically, uncertainty in NEO position grows one-dimensionally along its orbit with minimal radial and out-of-plane dispersions. This gives rise to the line of variations (LOV) concept, as quantified by Table 1's **`TCA3Sg`** column. Neglecting the relatively minor influences from an approached planet's gravity, **`TCA3Sg`** approximates the LOV's heliocentric arc-length in minutes of time to 3σ confidence. Thus, the LOV's 3σ length grows from the equivalent of heliocentric 2011 AG$_5$ motion during 0.25 min (0.00017 days) in 2011 to 16,066 min (11.157 days) in 2040.

At URL http://neo.jpl.nasa.gov/risk/2011ag5.html, NASA's Near Earth Object Program maintains a website from which the 1σ LOV semi-width w pertaining to any potential Earth impact can be obtained in units of Earth radii (r_E = 6378.136 km). With respect to 2011 AG$_5$ Earth approach on 5 February 2040, as predicted by JPL#45, $w = 0.00117\ r_E$ = 7.46 km. Projected onto a world map, this LOV would indeed appear to be a line when viewed at a global scale. The LOV projected in this manner is called a *risk corridor*.

---

[*] *Horizons* is accessible at URL http://ssd.jpl.nasa.gov/?horizons.

With $w \ll r_E$, any credible Earth impact is confined to the risk corridor, making this locus a valuable situational awareness tool. In utilizing this tool, it is essential to also maintain cognizance of what segment the risk corridor represents with respect to the entire LOV locus. The ratio of risk corridor length to LOV length approximates **`P_i/p`**. Consequently, Table 1 indicates credible 2011 AG$_5$ predictions actually comprising the 5 February 2040 risk corridor represent less than 0.16% of all credible predictions. As future 2011 AG$_5$ observations dictate the JPL#45 orbit solution be superseded, the most likely trend is for the 5σ LOV to contract and shift off the Earth entirely circa 5 February 2040. In accord with that trend, **`P_i/p`** will effectively reduce to zero, Earth impact will no longer be predicted in 2040, and no risk corridor will be associated with that Earth approach. Under less fortuitous (and less likely) circumstances, the LOV would shrink to *become* the risk corridor, **`P_i/p`** would increase to unity, and the LOV/risk corridor would ultimately contract to become the actual impact point on 5 February 2040.

Because 2011 AG$_5$ position uncertainty circa 2040 is well approximated by a LOV whose $w \ll r_E$, a method is documented and applied here to sample this LOV for Earth impact cases. On impact, defined to occur at a height of +42 km with respect to r_E, each case's geocentric position becomes a point on the risk corridor. Unlike a Monte Carlo analysis, which would randomly sample the JPL#45 uncertainty region for a multitude of credible 2011 AG$_5$ prediction cases (the vast majority of them missing the Earth), the technique documented here systematically and rapidly identifies the tiny subset of credible Earth impact cases confined to the JPL#45 LOV.

Earth Risk Corridor Computations for 2011 AG$_5$ on 5 February 2040

Risk Corridor Computations

With respect to nominal JPL#45 elements in Figure 1, a LOV dispersion is easily introduced with the equation $t' =$ `EPOCH` $+ \Delta t$. A user-specified small body (USSB) is defined to *Horizons* using t' in place of Figure 1's Julian date (JD) `EPOCH = 2455644.5` CT value. This USSB otherwise retains remaining elements at their JPL#45 nominal values and effectively samples a point on the solution's LOV.

```
JPL/HORIZONS                      (2011 AG5)                 2011-Nov-18 19:26:35
Rec #:641578 (+COV)   Soln.date: 2011-Oct-15_00:50:11     # obs: 197 (256 days)

FK5/J2000.0 helio. ecliptic osc. elements (AU, DAYS, DEG, period=Julian yrs):

  EPOCH=  2455644.5 ! 2011-Mar-24.00 (CT)         Residual RMS= .42895
    EC= .3906371176786754  QR= .8719004049292063  TP= 2455644.816053564
    OM= 135.7205763703037  W= 53.48251523509158   IN= 3.680497765026678
    A= 1.430839373753392   MA= 359.8179970436742  ADIST= 1.989778342577579
    PER= 1.71157           N= .57586111           ANGMOM= .018941824
    DAN= .98381            DDN= 1.57971           L= 189.1465019
    B= 2.9572041                                  TP= 2011-Mar-24.3160536

Physical parameters (KM, SEC, rotational period in hours):
    GM= n.a.               RAD= n.a.              ROTPER= n.a.
    H= 21.874              G= .150                B-V= n.a.
                           ALBEDO= n.a.           STYP= n.a.

ASTEROID comments:
1: soln ref.= JPL#45, PHA  OCC=4
2: source=ORB
```

Figure 1. Orbit elements for 2011 AG$_5$'s current JPL#45 solution appear as displayed by the *Horizons* telnet interface.

In the absence of all perturbations to conic heliocentric motion, USSB elements would match most of those in Figure 1 if coasted through $-\Delta t$ to `EPOCH`. The only deviations would be among those elements, such as `TP` (perihelion passage Julian date in CT) and `MA` (mean anomaly in deg), directly affected by sampling the LOV. But *Horizons* USSB orbit prediction accounts for all manner of perturbations to conic heliocentric motion, most notably gravitational accelerations from a dozen objects other than the Sun. Thus, if a USSB is created with Figure 1 elements using $\Delta t = +0.005041098$ days = +7.259181 min (the last case subsequently appearing in Table 4) and is coasted backward to `EPOCH`, its heliocentric semi-major axis will deviate from the Figure 1 value of `A` by +2.60385E-8 AU = +3.89530 km. With respect to a nominal orbit solution, the `EPOCH` bias Δt therefore introduces small velocity dispersions in addition to the intended position dispersions.

The `EPOCH`-biased USSB may then be processed by *Horizons* with capabilities applicable to any other catalogued NEO. In practice, t' is iterated using USSB approach tables until Earth **`CA Dist`** on 5 February 2040 is 0.000043 AU. This value corresponds to a perigee distance near 6400 km, close to a marginal or "grazing" impact. Generally, there are two grazing cases embedded in the LOV, one over Earth's leading limb in its heliocentric orbit and one over Earth's trailing limb. Table 2 documents the leading limb graze case's approach data, while Table 3 has the trailing limb graze case's approach data.

Because a USSB has no covariance data in *Horizons*, statistic parameters appearing in Table 1 are absent from Tables 2 and 3. Nevertheless, **`CA Dist`** values in Tables 2 and 3 can be compared with corresponding **`MinDist`** to **`MaxDist`** intervals in Table 1 as a necessary condition of plausibility for each grazing case. Only two minor deviations with respect to these intervals are evident. Each is associated with a grazing case's 2011 Earth approach only 25.36 days prior to JPL#45's `EPOCH`.

Table 2. A marginal impact case grazing Earth's leading limb is obtained by sampling the 2011 AG$_5$ LOV at Δt = +0.00504115 days. Corresponding planetary approach data are from the *Horizons* telnet interface.

```
                              AU      km/s
   Date (CT)        Body    CA Dist   Vrel
-----------------  -----  --------  ------
2011 Feb 26.66503  Earth  0.095723   7.516
2016 Sep 02.27817  Mars   0.067195  11.607
2023 Feb 03.35304  Earth  0.012289   9.922
2040 Feb 05.15392  Moon   0.002488  12.700
2040 Feb 05.15896  Earth  0.000043  14.671
```

Table 3. A marginal impact case grazing Earth's trailing limb is obtained by sampling the 2011 AG$_5$ LOV at Δt = +0.00490636 days. Corresponding planetary approach data are from the *Horizons* telnet interface.

```
                              AU      km/s
   Date (CT)        Body    CA Dist   Vrel
-----------------  -----  --------  ------
2011 Feb 26.66438  Earth  0.095722   7.516
2016 Sep 02.27813  Mars   0.067195  11.607
2023 Feb 03.35278  Earth  0.012291   9.922
2040 Feb 05.17354  Earth  0.000043  14.709
2040 Feb 05.56983  Moon   0.001335   9.353
```

Grazing cases in Tables 2 and 3 define the approximate termini of the 5 February 2040 risk corridor. These are not precise grazes because limited precision in `CA Dist` values may result in perigee heights departing from +42 km by ~100 km. To address this deficiency, *Horizons* is scripted to produce a geocentric USSB state vector at 5.0 February 2040 CT. A precision prediction accounting for Earth, Sun, and Moon gravity is initialized with this state vector at ~140,000 km geocentric distance and terminated ~4 hours later at +42 km height. Table 4 summarizes results of these USSB Earth impact cases. Impact CT is t_0. Geodetic latitude and longitude at impact are ϕ and λ, respectively. Speed at impact relative to a coordinate system rotating with the Earth is v_R, and inertial geocentric flight path angle at impact is γ.

Table 4. Earth impact cases on 5 February 2040 are sampled from the JPL#45 LOV and map the 2011 AG_5 risk corridor. Records in this table proceed from the trailing limb graze case to the leading limb graze case. The range in Δt spanned by these grazing cases is 0.00013522 days (0.19472 min).

Δt (days)	5 Feb CT t_0	ϕ (deg)	λ (deg)	v_R (km/s)	γ (deg)
+0.004905878	4:10:00.316	6.267 S	166.492 E	15.134	0.000
+0.0049068	4:08:10.776	5.061 S	179.404 E	15.129	-9.030
+0.004907	4:07:58.713	4.958 S	179.340 W	15.128	-9.920
+0.004908	4:07:05.935	4.568 S	174.102 W	15.122	-13.654
+0.00491	4:05:43.800	4.128 S	166.580 W	15.109	-19.061
+0.004915	4:03:07.874	3.772 S	153.624 W	15.079	-28.462
+0.00492	4:01:02.421	3.886 S	143.925 W	15.048	-35.534
+0.00493	3:57:35.800	4.799 S	128.546 W	14.987	-46.684
+0.00494	3:54:44.926	6.255 S	115.813 W	14.925	-55.639
+0.00495	3:52:18.919	8.076 S	104.465 W	14.863	-63.009
+0.00496	3:50:13.040	10.181 S	93.906 W	14.801	-68.621
+0.00497	3:48:24.942	12.529 S	83.757 W	14.739	-71.637
+0.00498	3:46:53.860	15.097 S	73.737 W	14.676	-71.080
+0.00499	3:45:39.836	17.880 S	63.549 W	14.613	-67.166
+0.00500	3:44:44.270	20.871 S	52.880 W	14.549	-60.967
+0.00501	3:44:09.877	24.080 S	41.224 W	14.485	-53.128
+0.00502	3:44:02.645	27.506 S	27.713 W	14.421	-43.618
+0.00503	3:44:37.407	31.100 S	10.259 W	14.357	-31.452
+0.005035	3:45:24.488	32.861 S	1.925 E	14.324	-23.248
+0.00504	3:47:14.502	34.137 S	22.759 E	14.292	-9.823
+0.005041098	3:48:54.161	33.432 S	38.575 E	14.285	0.000

The two grazing trajectories, whose impact parameters appear at the beginning and end of Table 4, are plotted with respect to a geocentric inertial coordinate system in Figure 2.

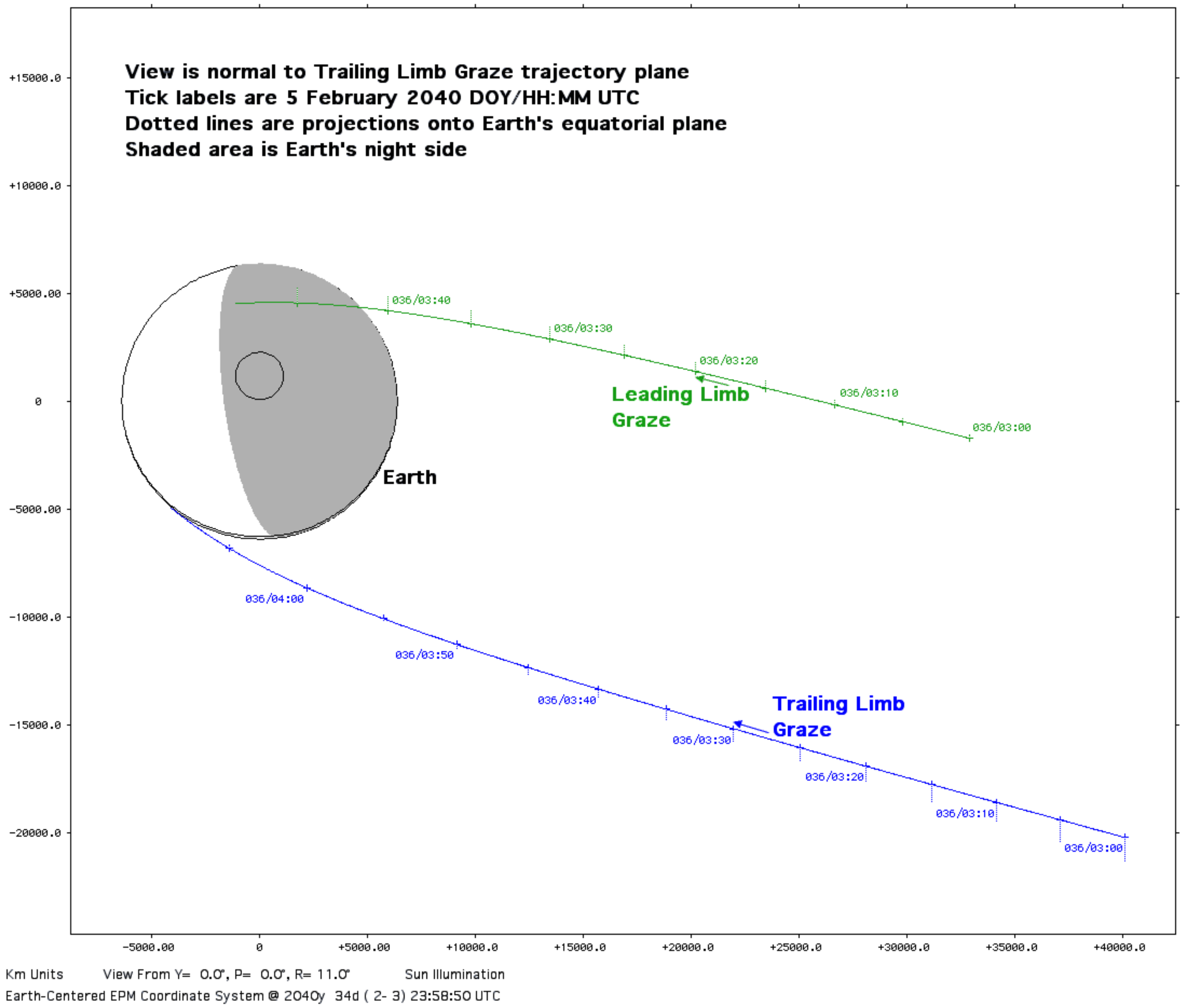

Figure 2. Earth's north rotational pole (as circumscribed by a small circle at 80° N latitude) appears just above the center of its disk in this inertial trajectory plot of grazing 2011 AG$_5$ trajectories whose perigee heights are +42 km. Both trajectory plots terminate at perigee. Heliocentric Earth motion is roughly toward the top of this plot in the general direction of the Leading Limb Graze trajectory's perigee. Because the Trailing Limb Graze trajectory approaches in a direction retrograde (opposed) to Earth's rotation, its v_R at the top of Table 4 is higher than any other impacting case.

Impact coordinates from Table 4 appear as "+" markers in the Figure 3 risk corridor plot.

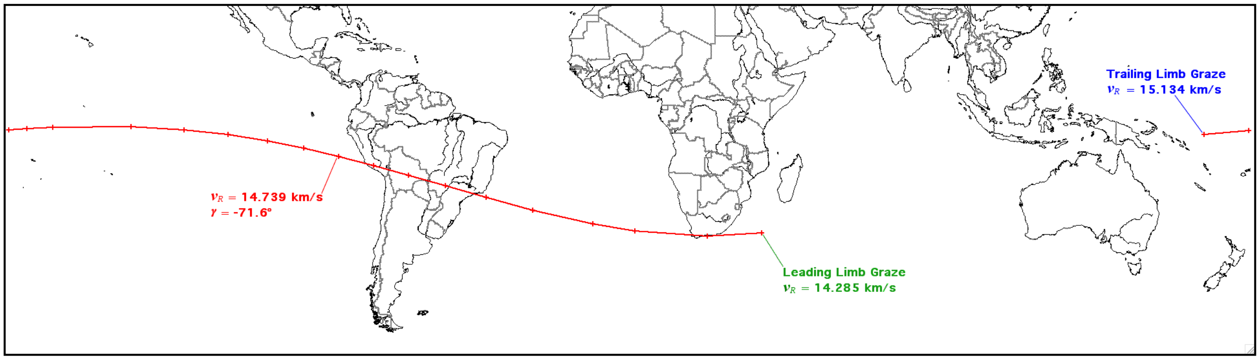

Figure 3. The risk corridor for 2011 AG$_5$ Earth impact on 5 February 2040 is plotted from LOV sampling associated with the JPL#45 orbit solution.

How plausible are LOV samples leading to the impact cases summarized in Table 4 and Figure 3? The Figure 1 orbit element whose uncertainty is most closely related to LOV arc-length expressed in time units is the time of perihelion `TP`. Although a 1σ `TP` uncertainty $TP\sigma$ is publicly unavailable at the JPL#45 epoch, a value can currently be obtained at an epoch 356 days later (14.0 March 2012 CT) via the JPL Small-Body Database Browser at URL http://ssd.jpl.nasa.gov/sbdb.cgi. With $TP\sigma = 0.0051255$ days from this source, Table 4 Δt values range from an equivalent $+0.957\sigma$ for the trailing limb graze to $+0.984\sigma$ for the leading limb graze. Therefore, impact cases sampled from the JPL#45 LOV appear plausible indeed.

An estimate of impact probability can be computed as the range of t_0 values in Table 4 (25.961 min) divided by the **`TCA3Sg`** = 16,066 min previously cited from Table 1 in association with the 5 February 2040 Earth approach. The quotient is 0.001616 (odds of 1-in-618.8), a value 3.8% greater than the corresponding Table 1 **`P_i/p`** = 0.001557. This is a strong indication that Δt values in Table 4 sample the JPL#45 LOV for Earth impacts with an accuracy sufficient to produce a meaningful risk corridor, as plotted in Figure 3.

Conclusion

In consultation with Solar System Dynamics personnel at JPL, the `EPOCH`-biased LOV sampling method documented here is thought to be valid for risk corridor computation purposes when **`P_i/p`** is sufficiently large. While this appears to be the case for the 2011 AG$_5$ Earth approach currently predicted in 2040, a minimum valid **`P_i/p`** threshold to associate with this method has yet to be determined. Additional risk corridor cases with **`P_i/p`** < 0.001 will be assessed with the method to explore its limitations. In any assessment with this method, the $w \ll r_E$ criterion will be a prerequisite.

An informal survey conducted by the author indicates the risk corridor is an overwhelmingly unfamiliar concept, even among astrodynamicists. This suggests a good deal of education regarding the pedigree and caveats of risk corridors is warranted without delay. A short-fuse hazardous NEO impact with Earth can develop at any time such that sheltering or evacuation in the threatened area is the only practical response. Risk corridor education *now* is preferable to attempting it during the fog of confusion under such circumstances. It is hoped this paper contributes to dispelling the risk corridor mystique among a broad audience.