International Space Station (ISS) Nears A Six-Figure Orbit Count

1. Introduction

Adopting the definition consistent with ubiquitous two-line element sets (TLEs) used to track virtually any artifact orbiting Earth, an orbit count n tallies the number of northbound equator crossings (ascending nodes), typically commencing with $n = 1$ at launch. Among the highest orbit counts ever achieved is that for Russia's long-lived *Mir* space station. With its original module launched on 20 February 1986, *Mir* was deorbited to undergo controlled disposal via incineration by Earth's atmosphere on 23 March 2001[i]. The final *Mir* TLE archived at https://www.space-track.org/ (accessed 24 June 2015) has $n = 86{,}330$. During its 5510-day flight history, *Mir* therefore logged an averaged mean motion $< \dot{M} > = 86{,}330 / 5510 = 15.668$ orbits per day.

Left to coast in orbit, satellites with proportionally large appendages like *Mir*'s solar arrays would suffer altitude decay from atmospheric drag and undergo uncontrolled atmospheric entry in a matter of months starting at a mean motion $\dot{M} > 15.6$ orbits per day. *Mir* stayed aloft for over 15 years because her mean motion was periodically reduced by prograde "reboost" maneuvers intended primarily for the purpose of prolonging orbit lifetime.

As a function of time, *Mir* $\dot{M}$ resembles a saw-tooth curve as atmospheric drag acts over many weeks to increase $\dot{M}$ and reboosts quickly reduce it. Over time, these two effects tend to balance each other. Consequently, initial Space Shuttle mission designs to achieve *Mir* rendezvous, conducted a year or more before planned launch, would simply coast a current *Mir* trajectory without atmospheric drag until the planned rendezvous timeframe. Although *Mir*'s location in its orbit would be highly uncertain a year in advance, *Mir* mean orbit height and its planar orientation would be reasonably accurate. This in turn permits reasonably accurate launch window times to be computed and reasonably accurate rendezvous propellant consumption to be estimated. Accuracy achieved versus *Mir*'s actual orbit on the planned launch date depended on the degree to which two factors were realized.

1) Other than its quasi-periodic saw-tooth variations, *Mir*'s actual $\dot{M}$ versus time curve had to be flat during the no-drag coast. *Mir* could not be in the process of transitioning to a higher or lower average orbit height during that interval. Likewise, a logistics breakdown or other departure from reboost planning could not result in a significant departure from initial average orbit height.

2) The initial *Mir* trajectory used to initiate the no-drag coast had to be representative of planned average orbit height during that interval. Typically, this entailed sampling the *Mir* trajectory about midway between reboosts.

On 2 September 2015 at 01:15:13 UT, ISS reached its $n = 96{,}000$ milestone after first element launch of Functional Cargo Block (FGB) *Zarya* on 20 November 1998[ii]. To achieve this

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[i] Reference https://en.wikipedia.org/wiki/Mir (accessed 30 August 2015).

[ii] Reference http://www.spaceref.com/iss/elements/fgb.html (accessed 30 August 2015).

milestone, ISS had $< \dot{M} > = 96{,}000 / 6130 = 15.661$ orbits per day, remarkably within 0.05% of the value spanning *Mir*'s flight history. This paper will apply the Space Shuttle rendezvous design technique to recent ISS TLEs and predict the range of times at which ISS $n = 100{,}000$ can be expected.

2. Method For Estimating Time At An Orbit Count Milestone

Several times per week, NASA posts recent and predicted TLEs spanning a two-week interval at http://spaceflight.nasa.gov/realdata/sightings/SSapplications/Post/JavaSSOP/orbit/ISS/SVPOST.html (accessed 16 September 2015). Table 1 contains an example TLE posted at this URL.

Table 1. An example TLE for ISS is reproduced as posted by NASA for public access. The three underlined values are relevant to computing orbit counts at a specified time.

```
1 25544U 98067A   15243.38161185  .00016717  00000-0  10270-3 0  9005
2 25544  51.6444  68.5781 0000859 163.5486 196.5696 15.55301747 39749
```

Underlined values in Table 1 are defined as follows.

1) Element set UT epoch $T_0 =$ `15243.38161185`: the value is expressed in `YYDOY.ffffffff` format, where `YY` is the last two digits of the calendar year, `DOY` is the ordinal day of the calendar year (1 January is `DOY` = 1; 31 December is `DOY` = 366 in a leap year), and `.ffffffff` is the UT day's decimal fraction. The value in Table 1 is equivalent to 31 August 2015 at 09:09:31 UT. Conceivably, T_0 may correspond to any location in the orbit, but NASA TLE postings for ISS compute T_0 immediately past a northbound equator crossing such that the decimal fraction of n is infinitesimal.

2) Mean motion $\dot{M} =$ `15.55301747` orbits per day: the value reflects an analytic compensation for Earth's excess equatorial mass. Ignoring other perturbations, including reboosts, n will increment by an amount very close to $\dot{M}$ every 24 hours.

3) Orbit count $n = 3974$: the TLE format accommodates an additional digit to the left of the Table 1 example, allowing values from 0 to 99,999 to be posted. But NASA's ISS operations are constrained to an interval $0 < n < 4001$, so the value posted at spaceflight.nasa.gov is modulo 4000 of the ascending nodes ISS has tallied since *Zarya* launch. On 2 September 2015 UT, NASA's ISS n incremented to 4000 before it cycled back to 1. Meanwhile, the value posted at space-track.org incremented from 96,000 to 96,001, where it will continue incrementing until it recycles from 99,999 to 0.

Given a milestone orbit count of interest n_M, the task is to compute the corresponding epoch T_M at which n increments to n_M per Equation 1 using underlined data from a single TLE per Table 1.

$$T_M = T_0 + (n_M - n) / \dot{M} \tag{1}$$

Equation 1 is simple to evaluate using modified Julian dates for T_0 and T_M, but accommodating equivalent calendar dates for these epochs on input and output can render it a bit more laborious to implement. Results from such an implementation are presented in Section 3 for ISS n_M = 100,000.

3. Predicted Times At Which ISS Orbit Count Will Reach 100,000

Beginning with the initial TLE of the ISS posting at spaceflight.nasa.gov on 17 June 2015 UT (and correcting posted n for modulo 4000 recycles), Equation 1 has been evaluated for n_M = 100,000 with the initial TLE of subsequent postings whenever practical. These evaluations are supplemented by others from space-track.org TLEs immediately before or after an ISS reboost. As illustrated by Figure 1, resulting T_M predictions all fall in a brief interval early on 16 May 2016 UT, verifying the Space Shuttle rendezvous design technique to a satisfactory degree.

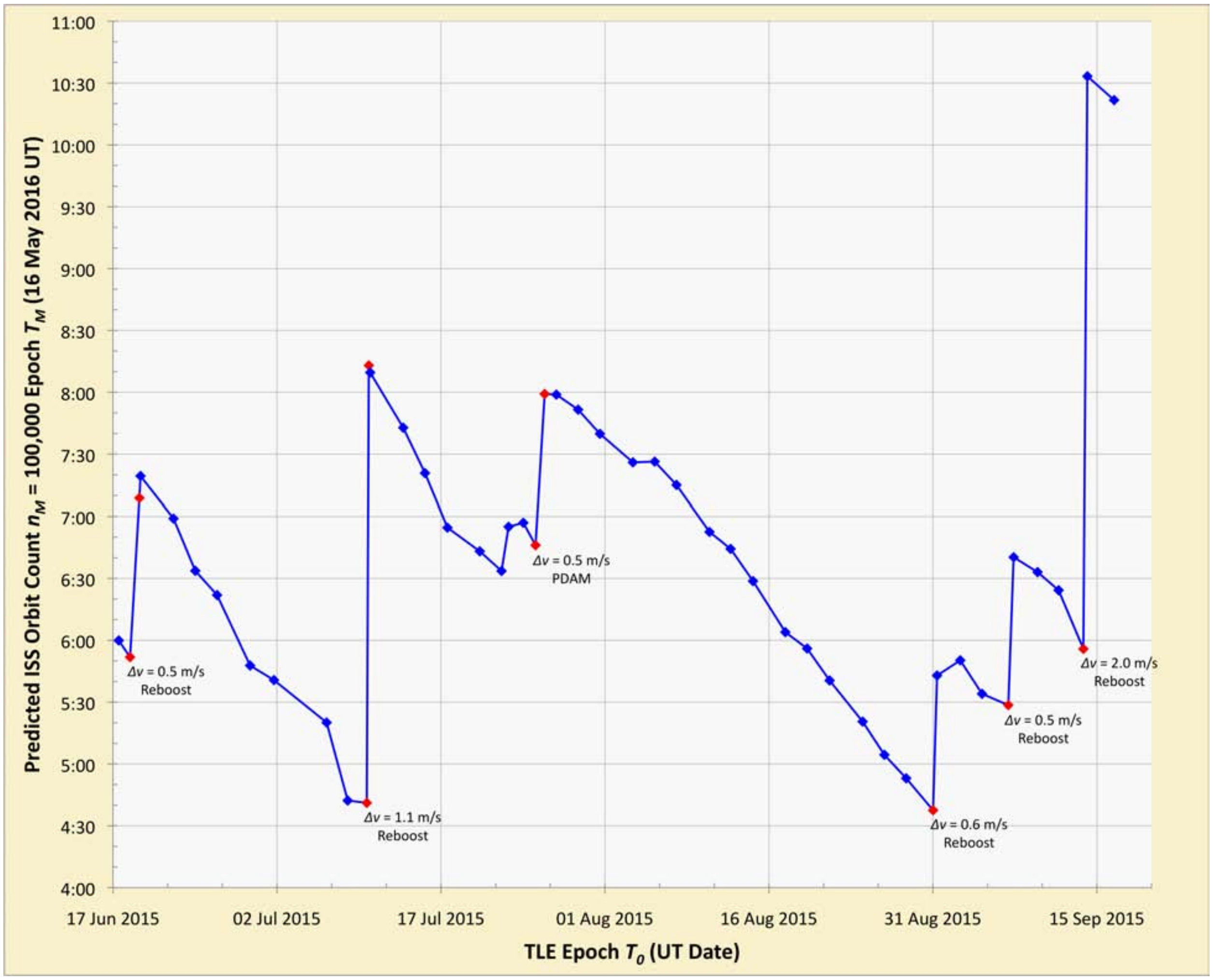


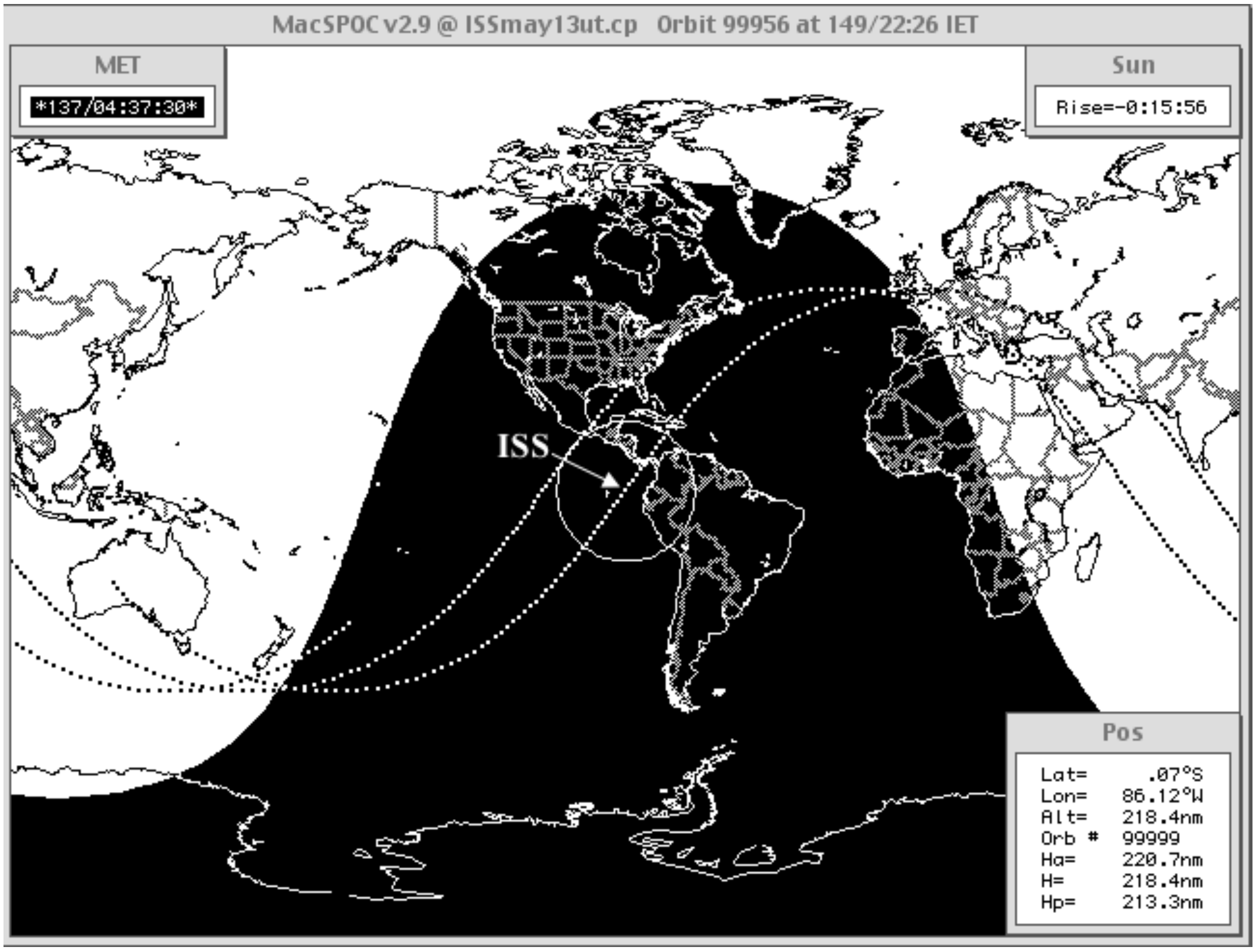
Figure 1. Predicted epoch T_M at which ISS orbit count will reach n_M = 100,000 is plotted as a function of TLE epoch T_0 assuming coasted no-drag motion between the two epochs. Delays (positive plot slope) in T_M are generally due to reboosts; advances (negative plot slope) in T_M are generally due to atmospheric drag. Thus, the plot's vertical scale may be regarded as roughly proportional to ISS orbit height. Red data markers are predictions derived from space-track.org TLEs, while blue data markers are predictions derived from spaceflight.nasa.gov TLEs.

Each data marker immediately prior to a Figure 1 reboost is annotated with the associated ISS change in velocity magnitude Δv. Unlike other planned reboosts appearing in Figure 1, the one performed 26 July UT is a Pre-determined Debris Avoidance Maneuver (PDAM). Unplanned PDAMs are typically executed a few times each year in response to an orbiting artifact whose probability of ISS collision is unacceptably high. Plans for subsequent reboosts must then be updated as required to compensate for the PDAM's unexpected effects.

In early August 2015, NASA released an ISS reference trajectory update extending well beyond May 2016. This planning product is coordinated among all ISS international partners, and it models both atmospheric drag and reboost orbit perturbations. The August 2015 reference trajectory predicts T_M for $n_M = 100{,}000$ will occur on 16 May 2016 at about 05:00 UT, confirming Figure 1 results are consistent with ISS trajectory planning.

4. Reconstruction Of The Actual n_M = 100,000 Milestone

As the ISS n_M = 100,000 milestone became historic fact, NASA posted bracketing geocentric Cartesian position/velocity state vectors to spaceflight.nasa.gov for epochs on 13 May 2016 at 09:28:33.804 UT (with n = 99,956) and on 16 May 2016 at 12:00:00.000 UT (with n = 100,004). A best-fit ISS ballistic atmospheric drag profile over this bracketing interval leads to an estimate of actual T_M =16 May 2016 at 04:37:34.837 UT, with a nadir longitude of 86.078° W. Residuals in semi-major axis and down-track position from the best-fit drag profile indicate the T_M estimate is in error by no more than ±0.008 s. Figure 2 illustrates the ISS ground track immediately before and after T_M.


MacSPOC v2.9 @ ISSmay13ut.cp   Orbit 99956 at 149/22:26 IET

MET
137/04:37:30

Sun
Rise=-0:15:56

Pos
Lat= .07°S
Lon= 86.12°W
Alt= 218.4nm
Orb # 99999
Ha= 220.7nm
H= 218.4nm
Hp= 213.3nm

Figure 2. The ISS ground track is plotted at 30-s intervals (2x2-pixel square markers), with the ISS nadir annotated between the Galapagos Islands and the coast of Ecuador, as the 100,000th orbit is about to begin. Circumscribing this nadir is a circle mapping the ISS horizon. The MET window displays UT in DOY/HH:MM:SS format, where DOY is the ordinal day of year 2016. Earth's nightside hemisphere, in which the annotated nadir is located, contains reverse-field pixels (white on black background).

The rightmost two data markers plotted in Figure 3 correspond to T_0 epochs bracketing T_M for $n_M = 100{,}000$, and its leftmost data markers span Figure 1's T_0 timescale. Note how effects on predicted T_M from aerodynamic drag and reboosts are reduced as T_0 approaches T_M. These "leverage" effects equate to reduced negative slopes between reboosts and smaller vertical jumps per unit reboost Δv as T_0 advances and these influences have less time prior to T_M in which to act. As an example of reduced reboost Δv leverage, compare the jump in T_M from the 0.5 m/s reboost in June 2015 with that from the 0.5 m/s reboost in April 2016. Leverage effects illustrate a fundamental trajectory design precept: for the sake of efficiency, schedule maneuvers and other trajectory control inputs as early as prediction uncertainties permit.

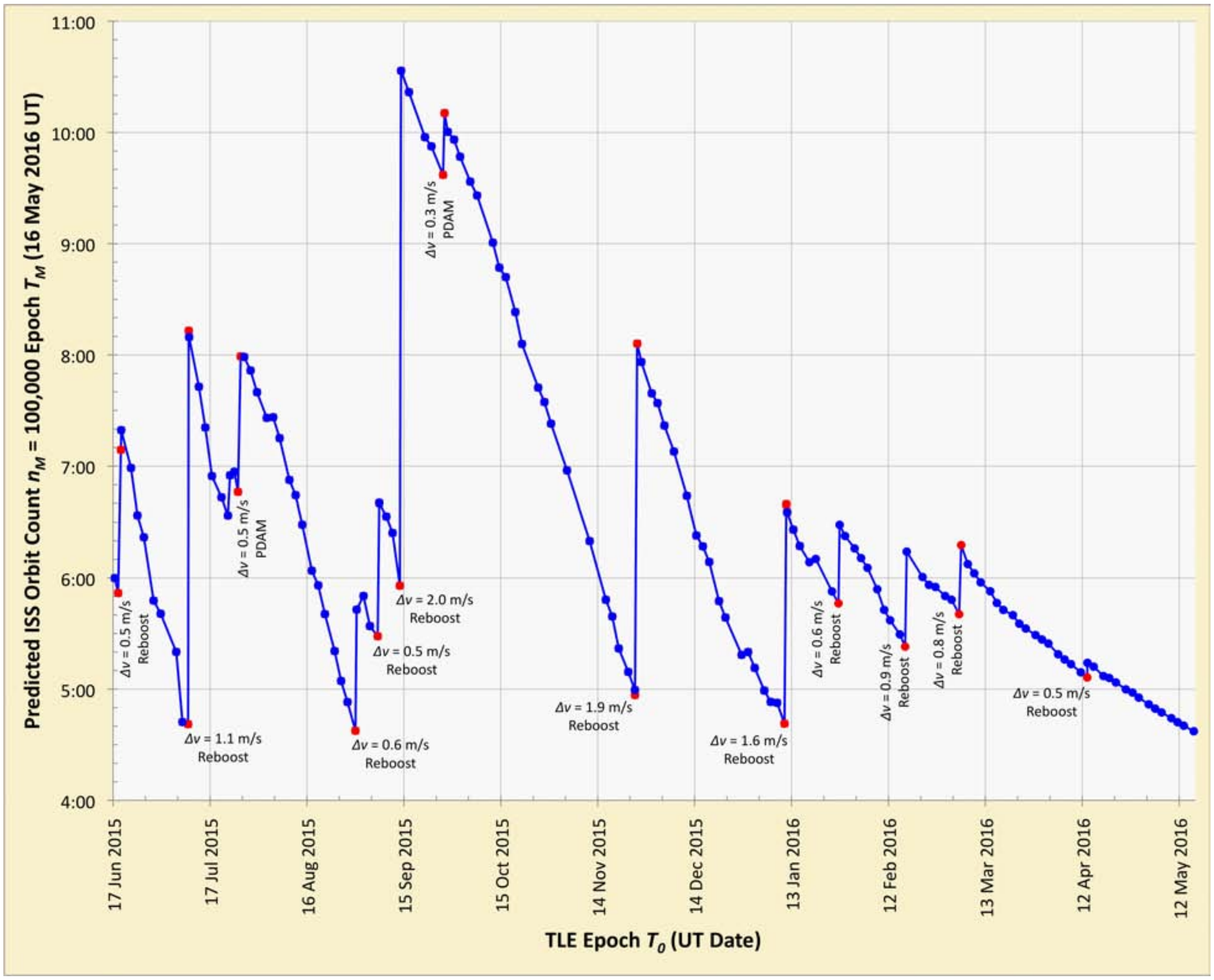


Figure 3. Predicted epoch T_M at which ISS orbit count will reach $n_M = 100{,}000$ is plotted as a function of TLE epoch T_0 assuming coasted no-drag motion between the two epochs. Red data markers are predictions derived from space-track.org TLEs, while blue data markers are predictions derived from spaceflight.nasa.gov TLEs.

Figure 3 also illustrates the degree to which ISS motion can be controlled via reboosts such that a long-term equilibrium is achieved with aerodynamic drag. If both drag and reboosts are ignored, their net effects impart variations in yearlong predictions amounting to less than 7 hours (about 4 orbits) of ISS motion.