A NEW DISCOVERY: NONLINEAR INSTABILITY LEADING TO LOSS OF CONTROL OF AIRCRAFT

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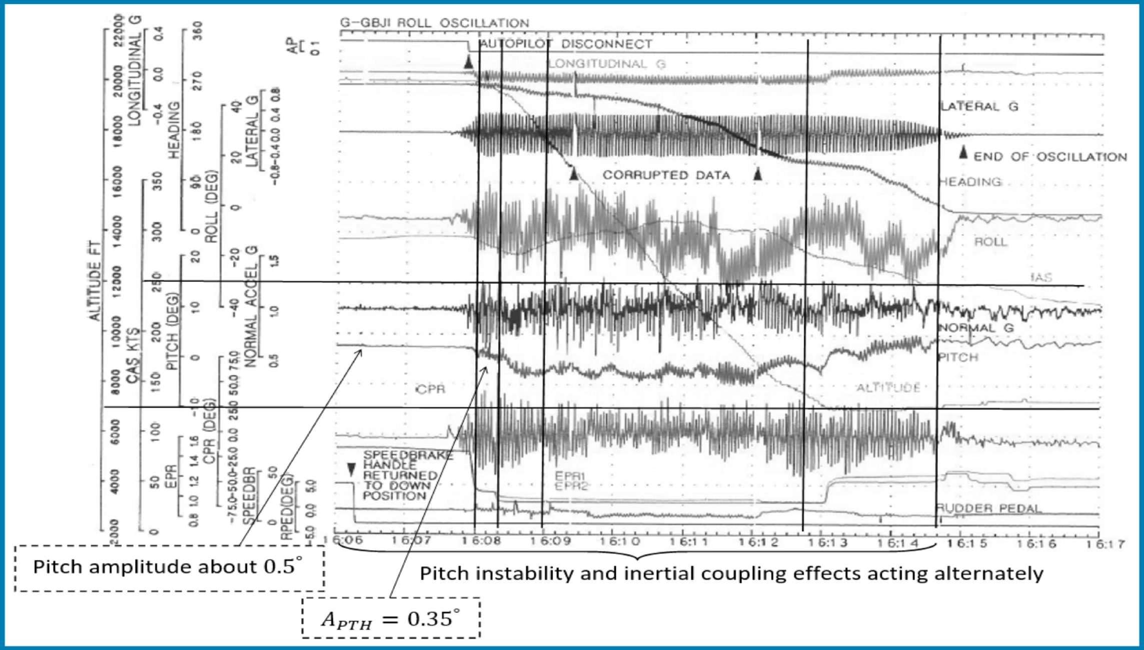


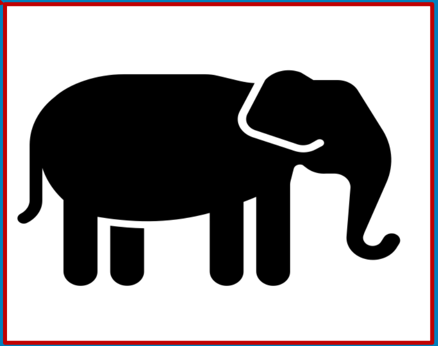
Neglected elephants in the equations

- The original governing equations for aircraft roll (φ), pitch (θ), and yaw (ψ) in the principal axis frame ($I_\varphi < I_\theta < I_\psi$):

$$
\begin{aligned}
I_\varphi \ddot{\varphi} + b_1 \dot{\varphi} + k_1 \varphi &= (I_\theta - I_\psi)\dot{\theta}\dot{\psi} + M_x(t) \\
I_\theta \ddot{\theta} + b_2 \dot{\theta} + k_2 \theta &= (I_\psi - I_\varphi)\dot{\varphi}\dot{\psi} + M_y(t) \\
I_\psi \ddot{\psi} + b_3 \dot{\psi} + k_3 \psi &= (I_\varphi - I_\theta)\dot{\varphi}\dot{\theta} + M_z(t)
\end{aligned}
$$

- The linearized equations:

$$
\begin{aligned}
I_x \ddot{\varphi} + b_1 \dot{\varphi} + k_1 \varphi &= M_x(t) \\
I_y \ddot{\theta} + b_2 \dot{\theta} + k_2 \theta &= M_y(t) \\
I_z \ddot{\psi} + b_3 \dot{\psi} + k_3 \psi &= M_z(t)
\end{aligned}
$$



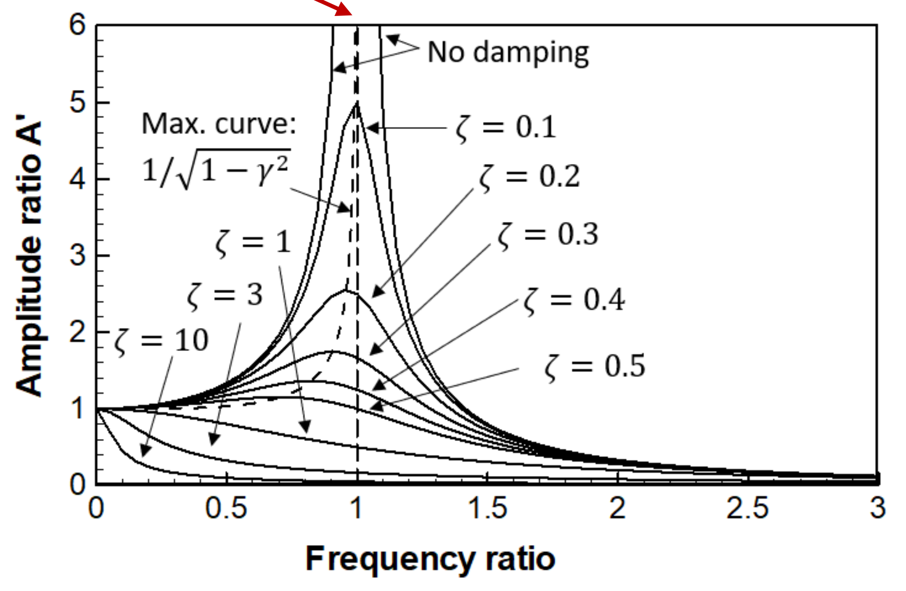
- $(I_y - I_z)\dot{\theta}\dot{\psi}$, $(I_z - I_x)\dot{\varphi}\dot{\psi}$, $(I_x - I_y)\dot{\varphi}\dot{\theta}$ - the inertial moments, similar to Coriolis force for hurricanes, should not be neglected

The elephants causing resonances

- As nonlinear harmonic oscillation system:

$$I_x\ddot{\varphi} + b_1\dot{\varphi} + k_1\varphi = \boxed{(I_y - I_z)\dot{\theta}\dot{\psi}} + M_x(t)$$
$$I_y\ddot{\theta} + b_2\dot{\theta} + k_2\theta = \boxed{(I_z - I_x)\dot{\varphi}\dot{\psi}} + M_y(t)$$
$$I_z\ddot{\psi} + b_3\dot{\psi} + k_3\psi = \boxed{(I_x - I_y)\dot{\varphi}\dot{\theta}} + M_z(t)$$

What matters the most is the exciting frequencies of these terms, not the amplitudes



A pitch control case

- The most common maneuver in takeoff, cruise leveling and landing approach

- The original governing equations:

$$I_x\ddot{\varphi} + b_1\dot{\varphi} + k_1\varphi = \left(I_y - I_z\right)\dot{\theta}\dot{\psi}$$

$$I_y\ddot{\theta} + b_2\dot{\theta} + k_2\theta = (I_z - I_x)\dot{\varphi}\dot{\psi} + M_{21}cos(\omega_{21}t + \alpha_{21})$$

$$I_z\ddot{\psi} + b_3\dot{\psi} + k_3\psi = \left(I_x - I_y\right)\dot{\varphi}\dot{\theta}$$

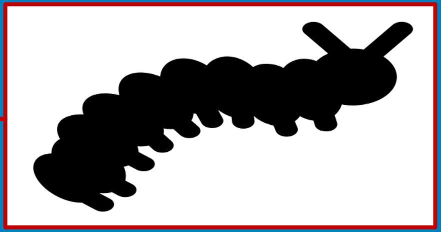
- The current longitudinal dynamics (pitch control only):

$$I_y\ddot{\theta} + b_2\dot{\theta} + k_2\theta = M_{21}cos(\omega_{21}t + \alpha_{21})$$

Solutions and bifurcations

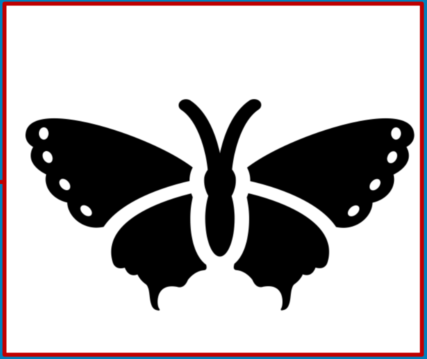
- By the current practice, the aircraft response is (caterpillar mode)

$$\left.\begin{aligned} \varphi &= 0 \\ \theta &= \theta_0(M_{21})\cos(\omega_{21}t + \alpha_{21} + \xi_{21}) \\ \psi &= 0 \end{aligned}\right\}$$



- This pitch-only response is not always stable. Under certain conditions, the following roll, pitch and yaw bifurcation solutions (butterfly mode) exist.

$$\left.\begin{aligned} \varphi(t) &= \sum_{i=1}^{\infty}\varphi_i(t) = \sum_{i=1}^{\infty} A_{1i}\cos(\omega_{1i}t + \beta_{1i}) \\ \theta(t) &= \sum_{j=1}^{\infty}\theta_j(t) = \sum_{j=1}^{\infty} A_{2j}\cos\left(\omega_{2j}t + \beta_{2j}\right) \\ \psi(t) &= \sum_{l=1}^{\infty}\psi_l(t) = \sum_{l=1}^{\infty} A_{3l}\cos(\omega_{3l}t + \beta_{3l}) \end{aligned}\right\}$$



Amplitudes of the 1st modes

- Solve the amplitudes:

$$A_{11} = \frac{2}{\omega_{11}}\left\{\frac{I_y I_z Z_{21} Z_{31}}{(I_z - I_x)(I_y - I_x)}\left[\frac{M_{21}}{2I_y Z_{21}}\left(\frac{(I_z - I_y)(I_y - I_x)}{I_x I_z Z_{11} Z_{31}}\right)^{1/2} - 1\right]\right\}^{1/2}$$

$$\boxed{A_{21} = \frac{2}{\omega_{21}}\left[\frac{I_x I_z Z_{11} Z_{31}}{(I_z - I_y)(I_y - I_x)}\right]^{1/2}}$$

Pitch amplitude cutoff from M_{21}

Pitch moment M_{21} driving roll and yaw

$$A_{31} = \frac{2}{\omega_{31}}\left\{\frac{I_x I_y Z_{21} Z_{11}}{(I_z - I_y)(I_z - I_x)}\left[\frac{M_{21}}{2I_y Z_{21}}\left(\frac{(I_z - I_y)(I_y - I_x)}{I_x I_z Z_{11} Z_{31}}\right)^{1/2} - 1\right]\right\}^{1/2}$$

Roll impedance: $Z_{11} = \sqrt{(\omega_{11}^2 - \omega_{10}^2)^2 + \left(\frac{b_1 \omega_{11}}{I_x}\right)^2}\,/\omega_{11}$

Yaw impedance: $Z_{31} = \sqrt{(\omega_{31}^2 - \omega_{30}^2)^2 + \left(\frac{b_3 \omega_{31}}{I_z}\right)^2}/\omega_{31}$

Pitch conditional instability criterion

- Pitch unstable if

$$M_{21} > \boxed{2I_y Z_{21}\left[\frac{I_x I_z Z_{11} Z_{31}}{(I_z - I_y)(I_y - I_x)}\right]^{1/2}}, \text{ pitch moment threshold}$$

$$\text{Pitch amplitude} > \boxed{A_{PTH} \equiv \frac{2}{\omega_{21}}\left[\frac{I_x I_z Z_{11} Z_{31}}{(I_z - I_y)(I_y - I_x)}\right]^{1/2}}, \text{ pitch threshold}$$

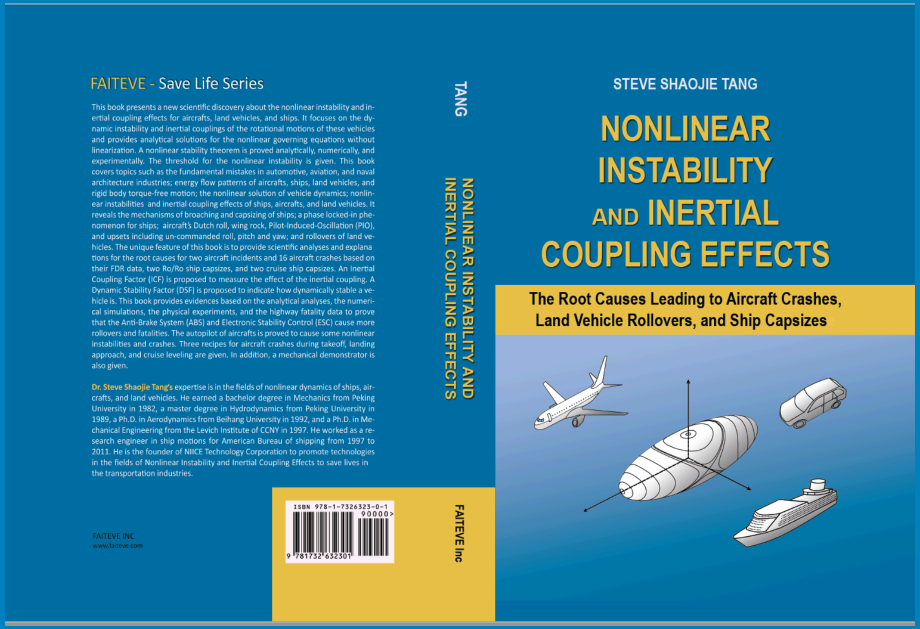
- Pitch stable if

$$M_{21} \le \boxed{2I_y Z_{21}\left[\frac{I_x I_z Z_{11} Z_{31}}{(I_z - I_y)(I_y - I_x)}\right]^{1/2}}$$

- Proved in the book,

  if the first modes $\varphi_1, \theta_1, \psi_1$ exist,

  $\varphi_2, \theta_2, \psi_2;\ \varphi_3, \theta_3, \psi_3, \ldots$ exist.



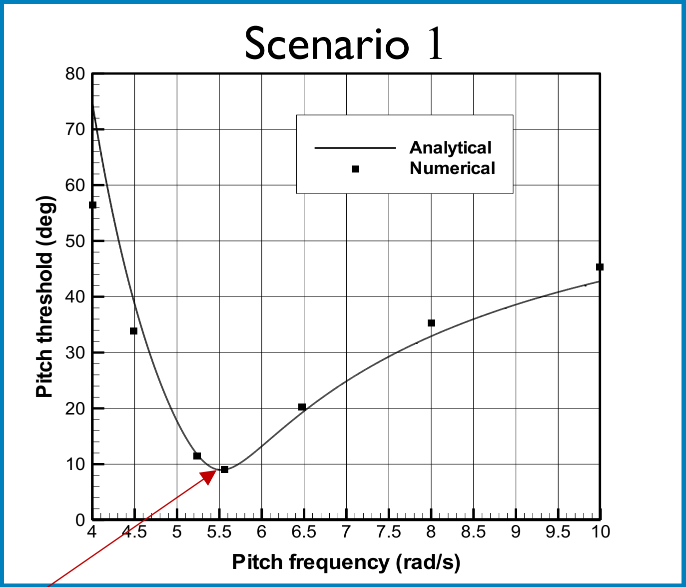
Pitch instability threshold verification

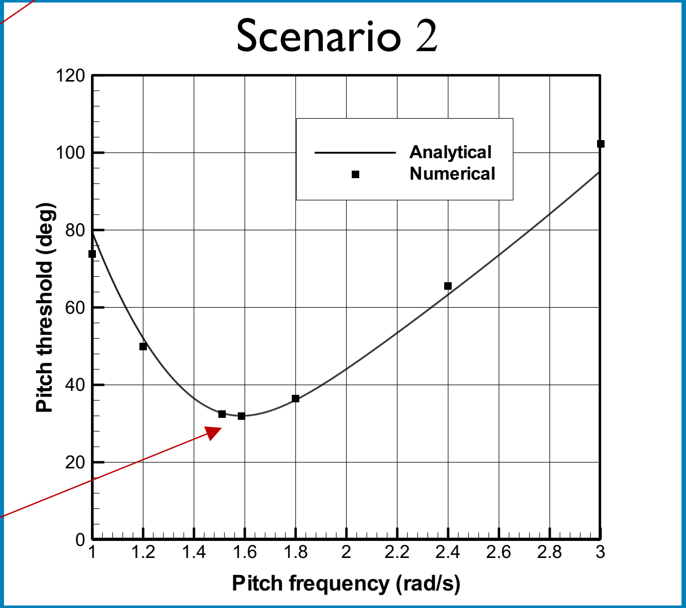
- A commercial aircraft model was used to do the numerical simulations.
- Pitch instability threshold in Scenario 1

The worst case: $\omega_{21} = \omega_{10} + \omega_{30}$

- Pitch instability threshold in Scenario 2

The worst case: $\omega_{21} = \omega_{10} - \omega_{30}$





The worst case – a resonance mode

- The pitch critical frequency:

$$\omega_{21} = \omega_{critical} \equiv \omega_{10} + \omega_{30}$$

- Pitch amplitude threshold

$$A_{PTH} \equiv \frac{2}{\omega_{10} + \omega_{30}} \left[\frac{b_1 b_3}{(I_z - I_y)(I_y - I_x)} \right]^{1/2}$$

- Two dangerous situations,

1. At stall,

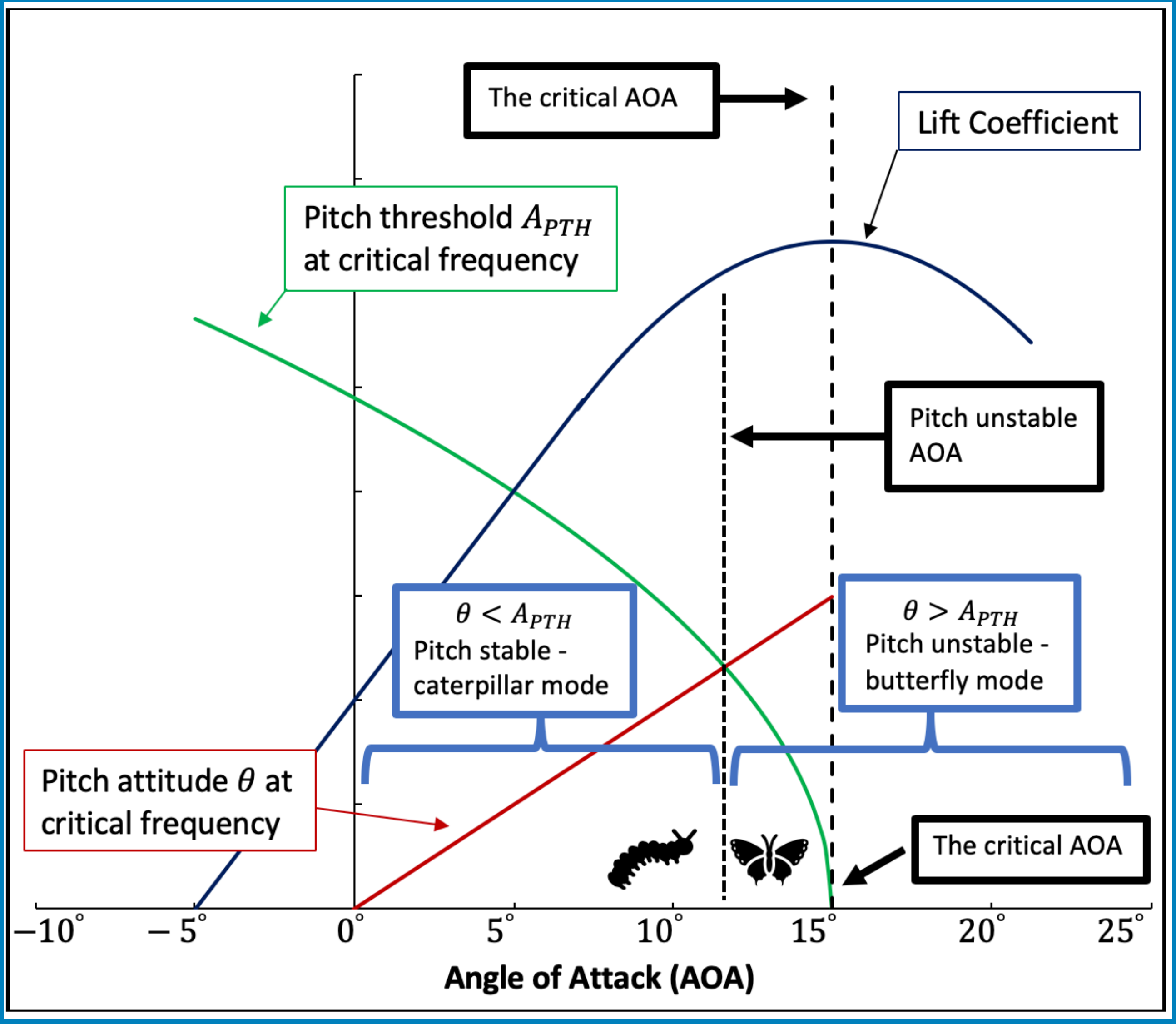
Roll damping; $b_1 \to 0$, then $A_{PTH} \to 0$

2. Yaw damper malfunction, turned off or hardover

Yaw damping: $b_3 \to 0$, then $A_{PTH} \to 0$

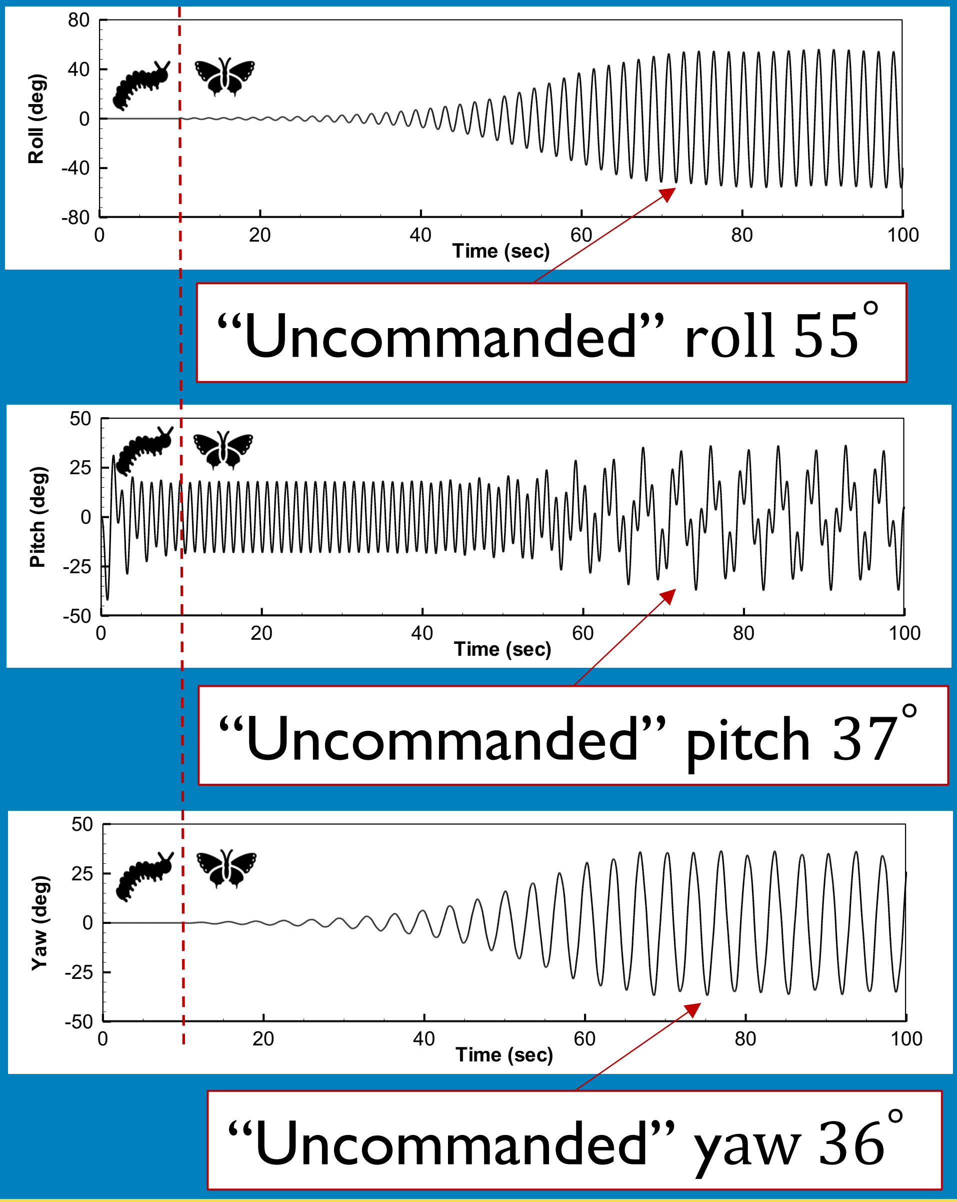
The danger during takeoff

- At the pitch critical frequency $\omega_{critical}$, aircraft is prone to nonlinear pitch instability before stall during takeoff



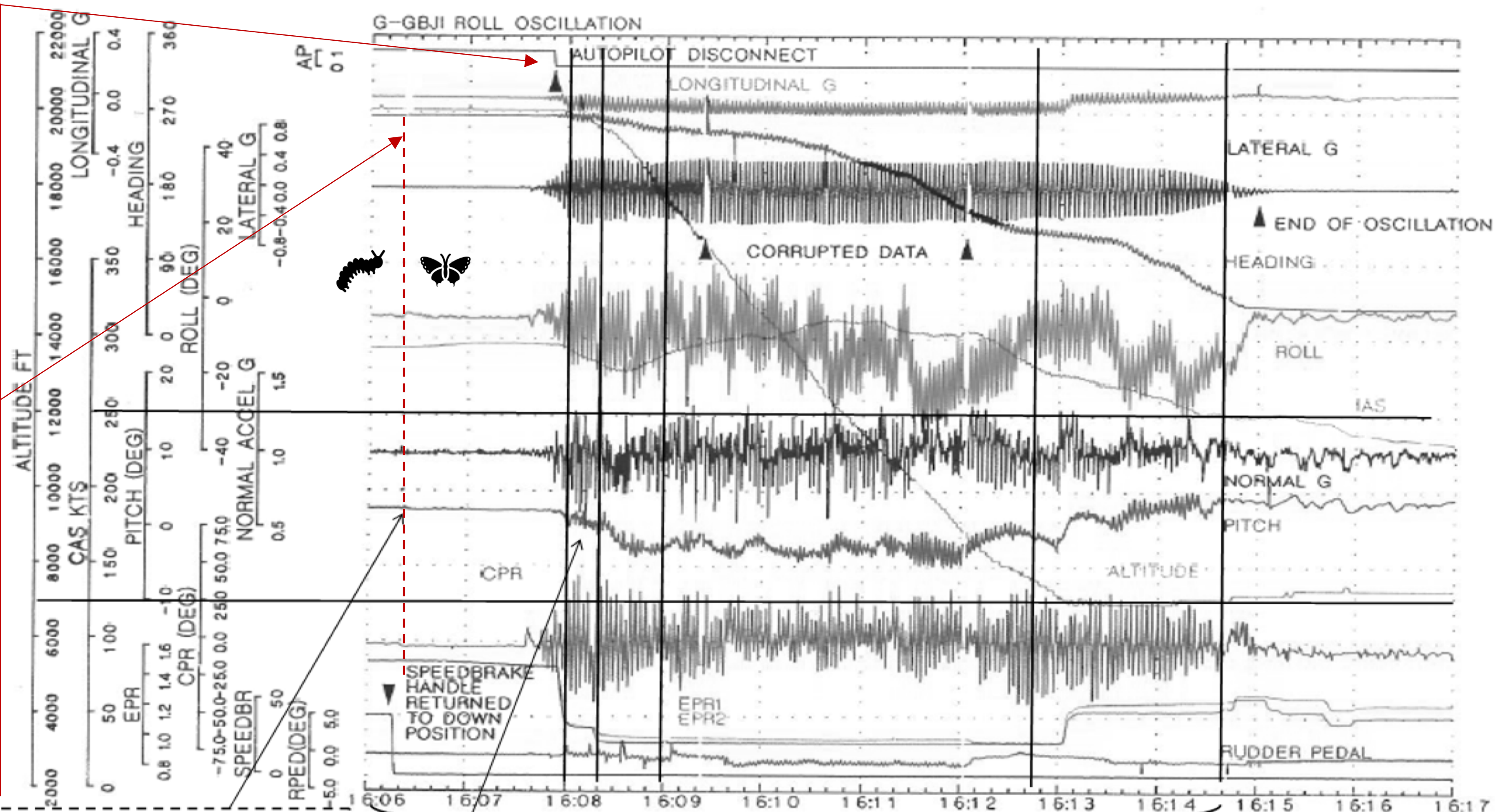
Loss of control of aircraft

- A pitch control producing 18° pitch oscillation at $\omega_{21} = 5.2\ (Rad/s)$
- The pitch amplitude threshold: $A_{PTH} = 12^{\circ}$
- Pitch unstable: pitch $18^{\circ} > A_{PTH} = 12^{\circ}$
- The final steady state amplitudes depend on the external pitch control amplitude



Nonlinear pitch instability leading to Loss of control of aircraft on B737-236 Advanced G-BGJI

B737-236 mode change from caterpillar to butterfly happened 1.5 min before autopilot disconnect

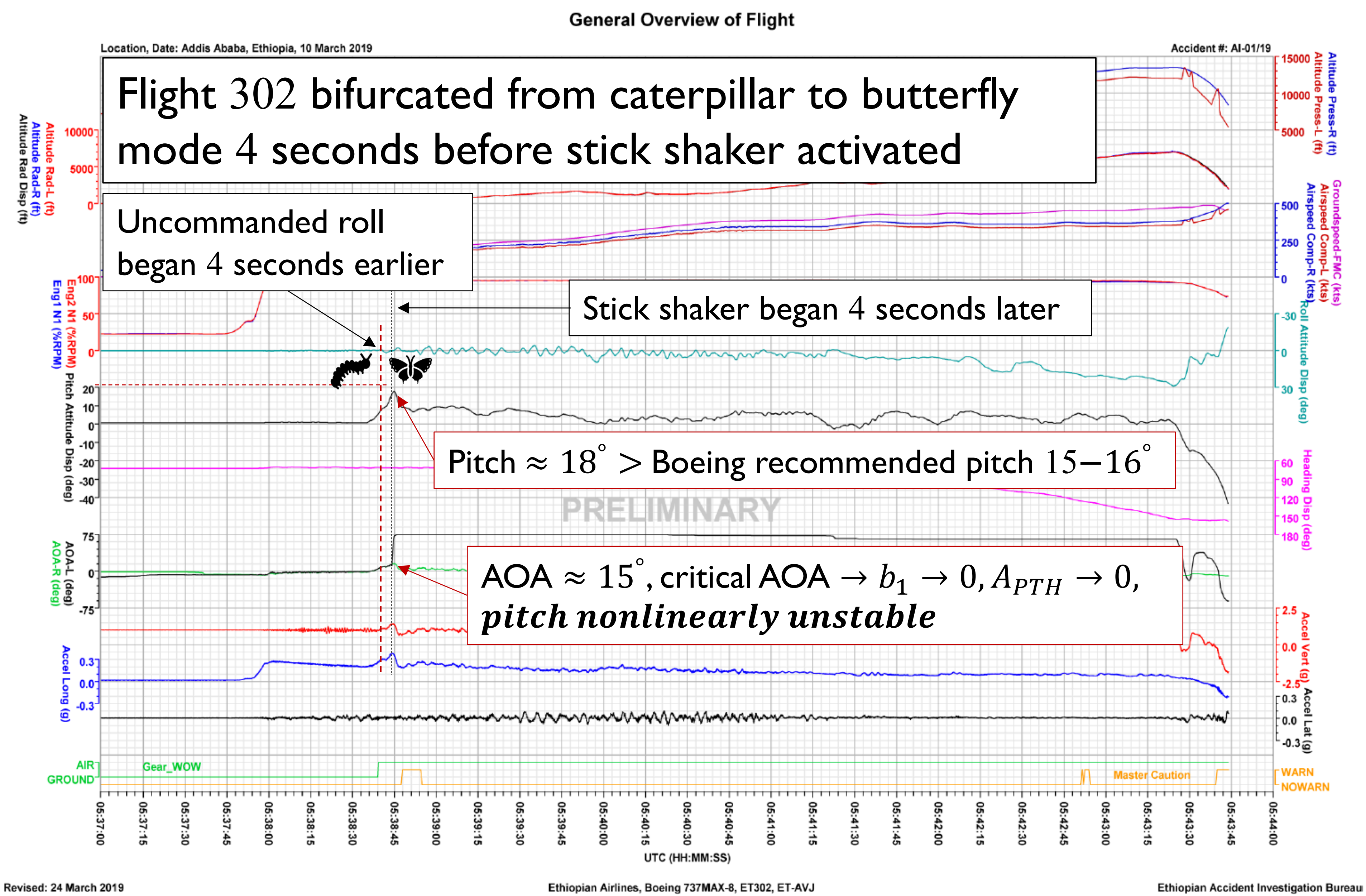


Pitch amplitude about 0.5°

Pitch instability and inertial coupling effects acting alternately

$A_{PTH} = 0.35^{\circ}$

Nonlinear pitch instability leading to Loss of control of aircraft on Ethiopian Airline Flight 302 B737 Max



The list of nonlinear pitch instability leading to loss of control of aircraft during takeoff

- Northwest Flight 255 MD DC-9-82 crashed in1987
- Delta Airlines Flight 1141 B727-232 crashed in1988
- USAir Flight 405 Fokker F-28 crashed in1992
- American Airline Flight 587 A300-605R crashed in 2001
- PT. Mandala Airlines Flight 091 B737-200 crashed in 2005
- Air Transat A310-308 C-GPAT upset in 2008
- Spainair Flight 5022 MD DC-9-82 crashed in 2008
- Gulfstream GVI (G650) N652GD test flight crashed in 2011
- Ethiopian Airlines Flight 302 B737 Max crashed in 2019

Nonlinear pitch instability demonstration

(www.youtube.com/watch?v=gG2-mu6I11A)

- Aircraft model with restoring and damping for roll and yaw

$$\omega_{10} = 2\pi, \quad T_{10} = 1\,sec$$
$$\omega_{30} = \pi, \quad T_{30} = 2\,sec$$

- The dangerous pitch frequencies

Scenario 1: $\omega_{21} = \omega_{10} + \omega_{30} = 3\pi$

$$\boxed{T_{21} = 0.7s,}\ A_{PTH-1} = \frac{2}{3\pi}\left[\frac{b_1 b_3}{(I_z - I_y)(I_y - I_x)}\right]^{1/2} = A_{PTH-2}/3$$

Scenario 2: $\omega_{21} = \omega_{10} - \omega_{30} = \pi$

$$\boxed{T_{21} = 2\,s,} \qquad A_{PTH-2} = \frac{2}{\pi}\left[\frac{b_1 b_3}{(I_z - I_y)(I_y - I_x)}\right]^{1/2}$$

- Experimental observation: $A_{PTH-2} = 3A_{PTH-1}$