Apollo 15 Trans-Earth Trajectory Reconstruction

1. Introduction

Even as Project Apollo milestones approach 50th anniversary commemorations, active research continues on scientific data collected during these historic missions to the Moon. Sensors in the Command-Service Module's (CSM's) Scientific Instrument Module (SIM) were amassing such data throughout Apollo 15's lunar orbit phase, and this activity continued after the trans-Earth injection (TEI) burn. Dr. Erik Kuulkers from the European Space Agency recently requested reconstruction of Apollo 15's as-flown post-TEI trajectory to support analysis of SIM data collected by the Gamma-Ray Spectrometer (GRS) and X-Ray Fluorescence Spectrometer (XRFS) instruments [1, p. 61]. With reconstructed Apollo 15 positions in the Earth-Moon system, Dr. Kuulkers hopes to constrain which astronomical sources could be associated with GRS and XRFS observations, particularly on 5 August 1971 [2, Appendix A]. This paper documents the reconstruction's method and results.

The reconstruction uses NASA Apollo Trajectory elements (NATs) to define perturbed Lambert boundary conditions spanning the interval from TEI cutoff on 4 August 1971 at 21:25:06.8 UT to ignition of the third midcourse correction actually performed (MCC-7) on 7 August 1971 at 17:30:49.9 UT[1]. The post-TEI trajectory is approximated with the WeavEncke predictor [4] operating at a fixed integration step of 120 s while modeling gravity accelerations from the Earth, Sun, and Moon as point masses. When in Earth's gravitational sphere of influence after 5 August 1971 at 14:19:06.8 UT, WeavEncke also models acceleration from Earth's excess equatorial mass. Velocity derived from TEI cutoff NATs is then differentially corrected to achieve NAT-derived position at MCC-7 ignition to better than 0.1 km precision. This process, previously documented in detail [5], ensures perturbations not modeled by WeavEncke are collectively approximated over the reconstructed trajectory.

Component parameters of NATs are defined as follows.

- GET ≡ ground elapsed time since Apollo 15 launch on 26 July 1971 at 13:34:00 UT in hours:minutes:seconds.
- h ≡ geodetic or selenocentric altitude in nmi.
- ϕ ≡ geodetic or selenocentric latitude in decimal deg, measured positively toward the north pole of planetary rotation.
- λ ≡ geodetic or selenocentric longitude in decimal deg, measured positively in the inertial direction of planetary rotation.
- s ≡ geocentric or selenocentric inertial speed in ft/s.
- γ ≡ flight path angle in decimal deg, measured positively above the local geocentric or selenocentric horizontal plane to inertial geocentric or selenocentric velocity.
- ψ ≡ inertial heading in decimal degrees, measured in the local geocentric or selenocentric horizontal plane from the projection of Earth's true north celestial pole to the projection of inertial geocentric or selenocentric velocity.

[1] Because Apollo 15 midcourse corrections were planned to be of zero magnitude, many were not executed. The planned midcourse correction in this timeframe is MCC-7 [3, p. 2-12, Table 2-9].

2. NAT Processing

With input from [6, p. 22, TABLE I], conversion of TEI cutoff NATs to selenocentric Cartesian position and velocity components in the Earth mean equator and equinox of epoch J2000.0 (J2K) is documented in Table 1. A similar conversion of MCC-7 ignition NATs to geocentric Cartesian J2K position and velocity is documented in Table 2.

Table 1. The as-flown Apollo 15 CSM selenocentric state vector at TEI cutoff is expressed using archived NATs [6, p. 22, TABLE I], together with equivalent J2K Cartesian position and velocity components.

Archived NATs	Equivalent J2K State Vector
GET = 223:51:06.8	4 August 1971 at 21:25:06.8 UT
h = +71.8 nmi	r_X = +770.268 km
ϕ = 18.30° S	r_Y = -1246.570 km
λ = 176.32° W	r_Z = -1162.555 km
s = 8272.4 ft/s	v_X = -2.095535978 km/s
γ = +4.43°	v_Y = -0.312155790 km/s
ψ = -129.08°	v_Z = -1.367071493 km/s

Table 2. The as-flown Apollo 15 CSM geocentric state vector at MCC-7 ignition is expressed using archived NATs [6, p. 22, TABLE I], together with equivalent J2K Cartesian position and velocity components.

Archived NATs	Equivalent J2K State Vector
GET = 291:56:49.9	7 August 1971 at 17:30:49.9 UT
h = +25,190.3 nmi	r_X = +32,512.865 km
ϕ = 38.43° S	r_Y = -25,982.827 km
λ = 102.64° E	r_Z = -32,850.772 km
s = 11,994.6 ft/s	v_X = -1.415139308 km/s
γ = -68.47°	v_Y = +2.804477944 km/s
ψ = +103.111°	v_Z = +1.870370134 km/s

3. Reconstructed Trajectory Generation

Epoch UTs and J2K positions from Table 1 and Table 2 serve as perturbed Lambert boundary conditions for the Apollo 15 trans-Earth trajectory reconstruction. Data in the two tables over-determine this reconstruction, so Table 1's J2K velocity components serve as a first approximation to their differential correction such that a WeavEncke coast achieves Table 2's J2K position components with a "miss" distance less than 0.1 km. The degree to which NATs are being properly interpreted and the coast is not overlooking important perturbations is indicated by the degree to which Table 1's J2K velocity components must be corrected to satisfy this miss goal. A coast of the uncorrected Table 1 J2K state vector misses Table 2 position by a distance of 762.505 km. After two differential correction iterations, however, miss distance is reduced to 0.000393 km by incorporating an aggregate Table 1 J2K velocity correction magnitude of 0.002604 km/s.

The corrected Table 1 J2K velocity is $\{-2.095902940, -0.312349353, -1.369642004\}^T$ km/s. This is effectively the initial velocity for a coast from TEI cutoff position extending to MCC-7 ignition over which the Apollo 15 CSM's actual as-flown trajectory is best approximated. Therefore, the corrected velocity and position from Table 1 are adopted as initial conditions for the desired trajectory reconstruction. When a coast from these initial conditions is performed, reconstruction validity is further confirmed by the vector difference magnitude between terminal geocentric J2K velocity and the NATs-derived J2K velocity from Table 2. This vector difference magnitude amounts to 0.005683 km/s and likely results primarily from CSM attitude maintenance and fluid dump accelerations not explicitly modeled during the WeavEncke coast. Note this difference with respect to Table 2 velocity has a magnitude of the same order as the aggregate magnitude of differential velocity corrections to Table 1 velocity.

Selenocentric inertial motion in the reconstructed trajectory immediately following TEI is plotted in Figure 1. Geocentric inertial motion for the entire reconstruction appears in Figure 2, together with geocentric inertial motion of the Moon during that interval. Time tick annotations in these plots imbed the UT date in a day-of-year (DOY) subfield for which 1 January is DOY = 1, and 31 December is DOY = 366 in a leap year.

Apollo 15 Trans-Earth Trajectory Reconstruction

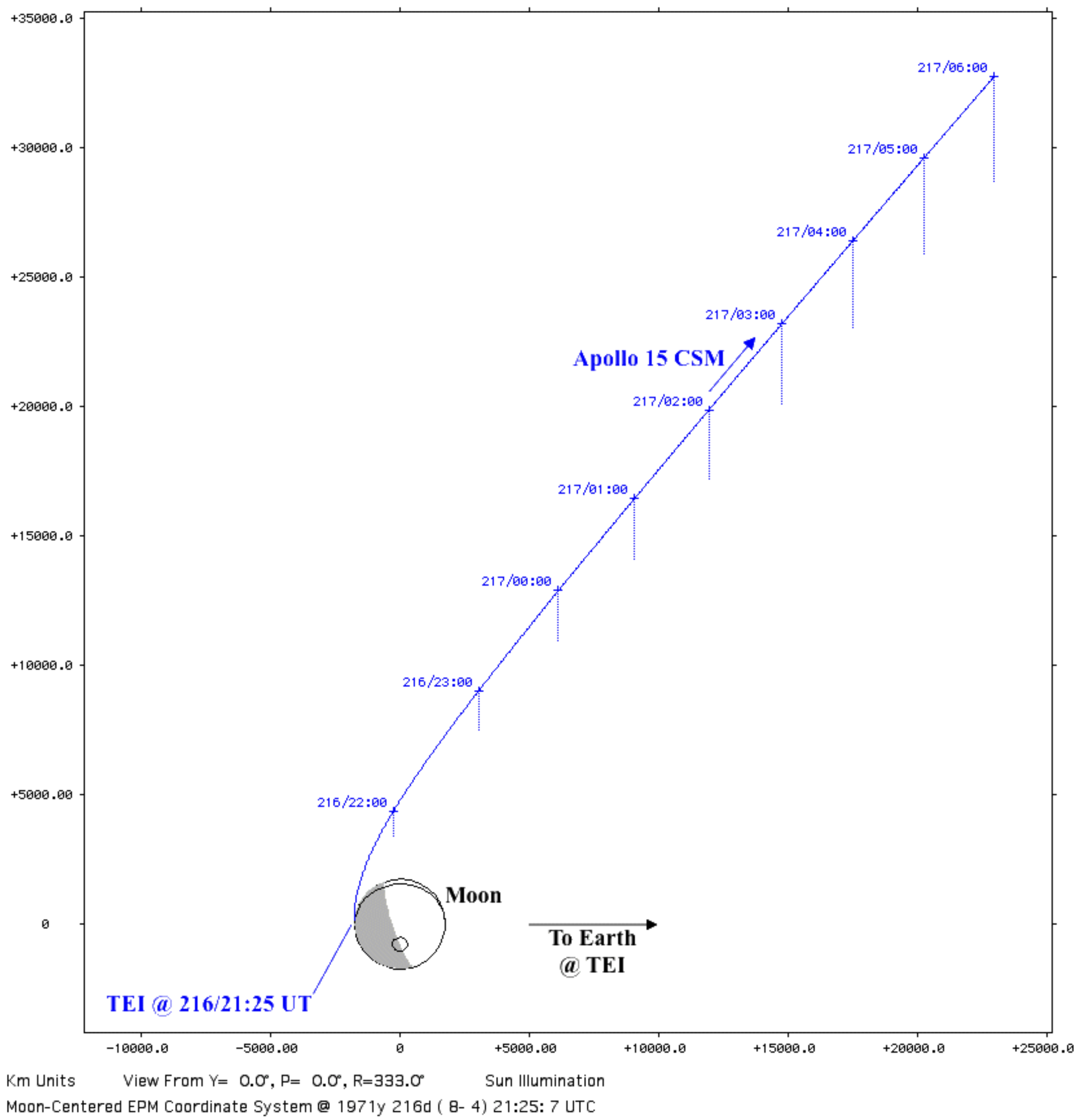

Figure 1. Selenocentric inertial motion in the reconstructed Apollo 15 trajectory is plotted immediately following TEI. Hourly time ticks are annotated with UT in DOY/hh:mm format during portions of 4 and 5 August 1971, and dotted lines are projections from each time tick onto the Moon's equatorial plane. The Moon's shaded area is its nightside.

Apollo 15 Trans-Earth Trajectory Reconstruction

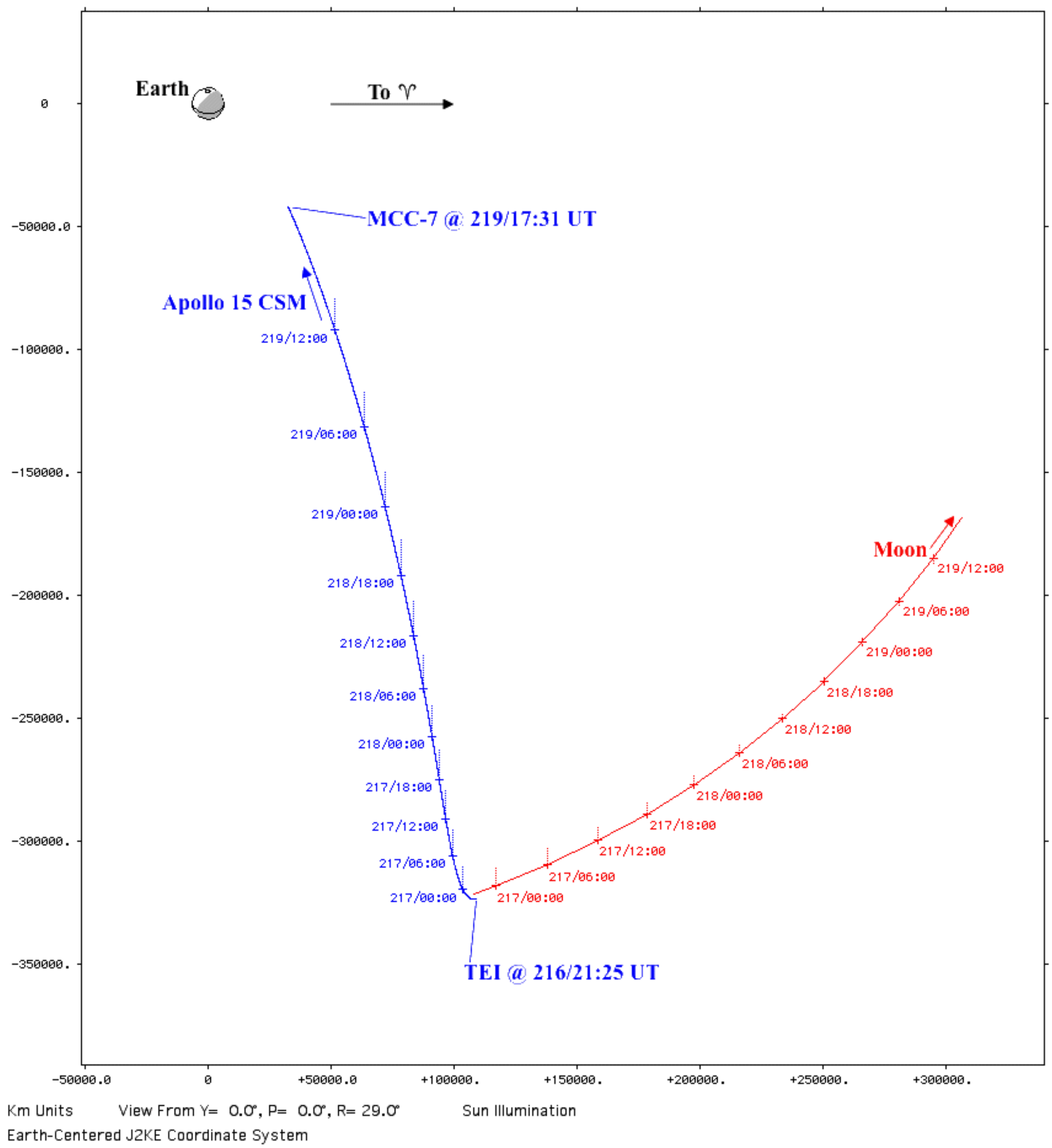

Figure 2. Geocentric inertial motion is plotted for the entire reconstructed Apollo 15 trajectory, together with that for the Moon during this interval. Time ticks at 6-hour intervals are annotated with UT in DOY/hh:mm format from 4 into 7 August 1971, and dotted lines are projections from each time tick onto the ecliptic plane. Earth's shaded area is its nightside.

4. Reconstructed Trajectory Verification

Apollo 15's SIM-based scientific observations were documented and preserved independently from flight operations data generally lost to posterity. Consequently, limited Apollo 15 as-flown trajectory data have been archived in association with GRS[2] and XRFS[3]. Because these archives are in binary format, Dr. Kuulkers kindly provided equivalent data in human-readable form. These data present a rare opportunity to spot-check the NATs-based trajectory reconstruction technique with an independent authoritative source.

Each spot-check entails a dedicated coast from reconstructed TEI cutoff conditions to the spot-check epoch where archived data exist. The coasted J2K Cartesian position is then transformed to components consistent with archived data. Deviations between the reconstructed and archived positions are quantified with three metrics. "Radial" deviation is the difference in position vector magnitudes in the sense "reconstructed minus archived". "Angular" deviation is the angle between the two position vectors. And "Vector" deviation is the vector difference magnitude between the two positions.

Archived position data for GRS are assumed to be geocentric Cartesian components in the Earth mean equator and equinox of Besselian epoch B1972.0 in accord with Apollo Lunar Surface Experiment Package (ALSEP) ephemeris convention [7, p. 2-3]. These positions extend from 4 August 1971 at 22:20:04.04 UT (about an hour after TEI) to 7 August 1971 at 13:34:22.22 UT (about 4 hours before MCC-7). Deviations from the GRS data appear in Table 3 and include spot-checks at the archived dataset's first and last epochs, together with an epoch about midway between them.

Table 3. Reconstructed geocentric position deviations from archived GRS data are spot-checked during trans-Earth coast.

1971 UT	Radial (km)	Angular (deg)	Vector (km)
4 Aug @ 22:20:04.04	-2.378	0.000774	5.587
6 Aug @ 05:57:11.11	+22.663	0.027624	133.376
7 Aug @ 13:34:22.22	-1.310	0.108505	177.679

Archived position data for XRFS extend from 4 August 1971 at 19:56:01.01 UT (about 1.5 hours before TEI) until 5 August 1971 at 11:48:00.00 UT (about 2.5 hours before the reconstructed WeavEncke coast departs the Moon's gravitational sphere of influence). During this interval, selenographic latitude and longitude, together with selenocentric distance, are archived. Deviations from the XRFS data appear in Table 4 and include spot-checks at the archived dataset's first post-TEI epoch, its last epoch with position data, and at an epoch midway between the other two.

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[2] GRS data archives may be downloaded at http://spdf.sci.gsfc.nasa.gov/pub/data/apollo/apollo15_csm/gamma-ray_spectrometer/gamma-ray_spectrometer_merge_data/ (accessed 11 March 2016).
[3] XRFS data archives may be downloaded at http://spdf.sci.gsfc.nasa.gov/pub/data/apollo/apollo15_csm/x-ray_fluorescence/transearth_coast_x-ray_data/ (accessed 11 March 2016).

Table 4. Reconstructed selenocentric position deviations from archived XRFS data are spot-checked during trans-Earth coast.

1971 UT	Radial (km)	Angular (deg)	Vector (km)
4 Aug @ 21:26:02.02	-2.901	0.016820	2.953
5 Aug @ 04:37:00.00	+9.547	0.051997	32.907
5 Aug @ 11:48:00.00	+17.514	0.054311	63.438

5. Conclusion

The utility of trajectory reconstruction based on NATs has been demonstrated in the context of ongoing scientific research enabled by observations from the Apollo 15 spacecraft during trans-Earth coast. Independent verification of this reconstruction indicates positions are accurate to better than 200 km.

References

[1] K. Atchison and H. Allaway, "Apollo 15 Press Kit", NASA Release 71-119K, 1971.[4]
[2] E. Kuulkers, J. J. M. in 't Zand, J. P. Lasota, "Restless Quiescence: Thermonuclear Flashes Between Transient X-Ray Outbursts", *Astronomy & Astrophysics*, Vol. 503, No. 3, pp. 889-897, 2009.[5]
[3] S. H. Gardner, "Apollo 15 Flight Plan", NASA-MSC, 1971.[6]
[4] D. R. Adamo, "A Precision Orbit Predictor Optimized for Complex Trajectory Operations", AAS 03-665, *Volume 116 of the Advances in the Astronautical Sciences*, Univelt, San Diego, pp. 2567-2586, 2003.
[5] D. R. Adamo, "Apollo 13 Trajectory Reconstruction via State Transition Matrices", *Journal of Guidance, Control, and Dynamics*, Vol. 31, No. 6, AIAA, Washington, D.C., pp. 1772-1781, 2008.
[6] J. R. Gurley and J. P. Mayer, "Postflight Evaluation of the Apollo 15 Spacecraft Trajectories", NASA MSC Internal Note No, 71-FM-420, 1972.[7]
[7] W. W. Lauderdale and W. F. Eichelman, "Apollo Scientific Experiments Data Handbook", JSC-09166, NASA TM X-58131, 1974.

[4] This document is available for download at http://history.nasa.gov/alsj/a15/A15_PressKit.pdf (accessed 4 March 2016).

[5] This paper is available for download at http://www.aanda.org/component/article?access=bibcode&bibcode=&bibcode=2009A%2526A...503..889KFUL (accessed 11 March 2016).

[6] This document is available for download at http://www.ibiblio.org/apollo/Documents/Apollo15FlightPlan.pdf (accessed 8 March 2016).

[7] This 49-page document has been scanned into the JSC History Collection at the University of Houston-Clear Lake. To obtain an electronic copy free of charge, contact the collection's Archivist via http://www.jsc.nasa.gov/history/history_collection/uhcl.htm (accessed 5 March 2016) and request HSI-44563.pdf.