New Approach for Real-Time Inertia Estimation

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Outline

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- Procedure
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Problem Definition

- Goal?

  - Estimate moment of inertia (MOI) of S/C in real-time

- MOI of SC is unknown?

  - Known before launch but it will be changed by: sloshing, consumption of propellant, deployment of solar panel, etc.

- Why does MOI info. need?

  - For precise attitude control of S/C.

- Relation b/w control ($\boldsymbol{u}$) and MOI of S/C (J)?

$$\boldsymbol{u}(t) = -K(J)\boldsymbol{q}_{err} - D(J)\boldsymbol{\omega}_{err} = \boldsymbol{f}(J, \boldsymbol{q}, \boldsymbol{\omega})$$

where $\boldsymbol{q}$ is the attitude and $\boldsymbol{\omega}$ is the angular velocity.

Procedure

A. Filtering: Extended Kalman Filter (EKF)
   - for measured angular velocity including noise.

B. Calculating: Savitzky-Golay Filter (SGF)
   - for angular acceleration given measured angular velocity.

C. Estimating: Recursive Least Squares (RLS)
   - for MOI of S/C acquisition.

Equations of Motion (1/2)

Dynamic Equation

$$\dot{\boldsymbol{\omega}} = -J^{-1}[\boldsymbol{\omega}^{\times}]J\boldsymbol{\omega} + J^{-1}\mathbf{u} = \boldsymbol{f}(\boldsymbol{\omega}, \mathbf{u}, t)$$

where $\boldsymbol{\omega}^{\times}$ is the cross product of $\boldsymbol{\omega}$ and $J = \text{diag}\{J_1, J_2, J_3\}$.

$$\Rightarrow \quad \boxed{\dot{\mathbf{x}}(t) = \boldsymbol{f}(\mathbf{x}(t), \mathbf{u}(t), t) + \boldsymbol{w}(t)}$$

where $\mathbf{x}(t) \triangleq \boldsymbol{\omega}(t)$, $E\{\boldsymbol{w}(t)\} = 0$, and $E\{\boldsymbol{w}(t)\boldsymbol{w}(t)^T\} = Q(t)$.

Linearize $\boldsymbol{f}\left(\mathbf{x}(t), \mathbf{u}(t), t\right)$ neglecting H.O.T yields

$$\boldsymbol{f}(\mathbf{x}(t), \mathbf{u}(t), t) \cong \boldsymbol{f}(\hat{\mathbf{x}}(t), \mathbf{u}(t), t) + F\left(\hat{\mathbf{x}}(t), t\right)\left[\mathbf{x}(t) - \hat{\mathbf{x}}(t)\right]$$

where $F\left(\hat{\mathbf{x}}(t), t\right) \triangleq \left.\dfrac{\partial \boldsymbol{f}}{\partial \mathbf{x}}\right|_{\hat{\mathbf{x}}(t)}$.

Measurement Equation

$$\boldsymbol{y} = H\boldsymbol{x}(t) + \boldsymbol{v}(t)$$

where $E\{\boldsymbol{v}(t)\} = 0$ and $E\{\boldsymbol{v}(t)\boldsymbol{v}(t)^{\mathrm{T}}\} = R'(t)$. Also,

$$\boldsymbol{y} \triangleq [\tilde{\omega}_1 \quad \tilde{\omega}_2 \quad \tilde{\omega}_3]^{\mathrm{T}} = \text{measured value of angular velocity}$$

$$H \triangleq I_{3\times 3} = \text{observation matrix}$$

$$\boldsymbol{x} \triangleq [\omega_1 \quad \omega_2 \quad \omega_3]^{\mathrm{T}} = \text{true value of angular velocity}$$

$$\boldsymbol{v} \triangleq [v_1 \quad v_2 \quad v_3]^{\mathrm{T}} = \text{measurement error}$$

Extended Kalman Filter (EKF): Angular velocity

Time Update ("Predict")

1) Project the state ahead

$$\dot{\hat{x}}(t) = f\left(\hat{x}, u, t\right) + G(t)w(t)$$

2) Project the error covariance ahead

$$\dot{P} = FP + PF^{T} + GQG^{T}$$

Initial estimates for $\hat{x}(t_0)$ and P_0

Measurement Update ("Correct")

1) Compute the Kalman gain

$$K_k = P_k^- H_k^T \left(H_k P_k^- H_k^T + R_k\right)^{-1}$$

2) Update estimate with measurement z_k

$$\hat{x}_k^+ = \hat{x}_k^- + K_k\left[z_k - h_k\left(\hat{x}_k^-\right)\right]$$

3) Update the error covariance

$$P_k^+ = \left(I - K_k H_k\right) P_k^-$$

Filtering Techniques (2/3)

Savitzky-Golay Filter* (SGF): Angular acceleration

- ✓ 1st derivative estimation
- ✓ Conditions
  - fixed sample time interval
  - consecutive data
  - odd number
- ✓ Formula for 7 data points (classical)

$$y'_i = \frac{1}{28}(-3 \cdot y_{i-3} - 2 \cdot y_{i-2} - 1 \cdot y_{i-1} + 0 \cdot y_i + 1 \cdot y_{i+1} + 2 \cdot y_{i+2} + 3 \cdot y_{i+3})$$

Coefficient for quadratic form

Window size	3	5	7	$\cdots$	2n+1
n					n
$\vdots$				$⋰$	$\vdots$
-3			-3	$\cdots$	-3
-2		-2	-2	$\cdots$	-2
-1	-1	-1	-1	$\cdots$	-1
0	0	0	0	$\cdots$	0
1	1	1	1	$\cdots$	1
2		2	2	$\cdots$	2
3			3	$\cdots$	3
$\vdots$				$\ddots$	$\vdots$
n					n
Normalization	2	10	28	$\cdots$	$\sum_{-n}^{n} k^2$

* Savitzky, A. and Golay, M. J. E., "Smoothing and Differentiation of Data by Simplified Least Squares Procedures," *Analytical Chemistry*, Vol. 36, 1964, pp. 1627-1639.

Filtering Techniques (3/3)

Savitzky-Golay Filter (SGF): Angular acceleration

- ✓ Casual form for real time estimation ⟹Time lag

$$y'_i = \frac{1}{28}(-3 \cdot y_{i-6} - 2 \cdot y_{i-5} - 1 \cdot y_{i-4} + 0 \cdot y_{i-3} + 1 \cdot y_{i-2} + 2 \cdot y_{i-1} + 3 \cdot y_i)$$

- ✓ Trade-off

Window size	Advantage	Disadvantage
Large	accurate	Time lag Computational burden

- ✓ Adopt variable window size

Data	3	4	5	6	7 ~
Window size	3	3	5	5	7

  – First backward derivative for initial data

Estimation Method (1/2)

Recursive Least Squares (RLS): MOI of S/C

$$\boldsymbol{z} = H\boldsymbol{x} + \boldsymbol{v}$$

where $E\left\{\boldsymbol{v}(t)\right\} = 0$ and $E\left\{\boldsymbol{v}(t)\boldsymbol{v}(t)^{\mathrm{T}}\right\} = R(t)$. Also,

$$\boldsymbol{z} \triangleq \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix}^{\mathrm{T}} = \text{commanded control torque}$$

$$H \triangleq \begin{bmatrix} \dot{\omega}_1 & -\omega_2\omega_3 & \omega_2\omega_3 \\ \omega_1\omega_3 & \dot{\omega}_2 & -\omega_1\omega_3 \\ -\omega_1\omega_2 & \omega_1\omega_2 & \dot{\omega}_3 \end{bmatrix} = \text{observation matrix}$$

$$\boldsymbol{x} \triangleq \begin{bmatrix} J_1 & J_2 & J_3 \end{bmatrix}^{\mathrm{T}} = \text{unknown true MOI of S/C}$$

$$\boldsymbol{v} \triangleq \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix}^{\mathrm{T}} = \text{measurement error}$$

Recursive Least Squares (RLS): MOI of S/C

$$\hat{x}_{k+1} = \hat{x}_k + K_{k+1}\left(y_{k+1} - H_{k+1}\hat{x}_k\right)$$

where

$$K_{k+1} = P_k H_{k+1}^{\mathrm{T}}\left(H_{k+1} P_k H_{k+1}^{\mathrm{T}} + R_{k+1}\right)^{-1}$$

$$P_{k+1} = \left(I - K_{k+1}H_{k+1}\right)P_k\left(I - K_{k+1}H_{k+1}\right)^{\mathrm{T}} + K_{k+1}R_{k+1}K_{k+1}^{\mathrm{T}}$$

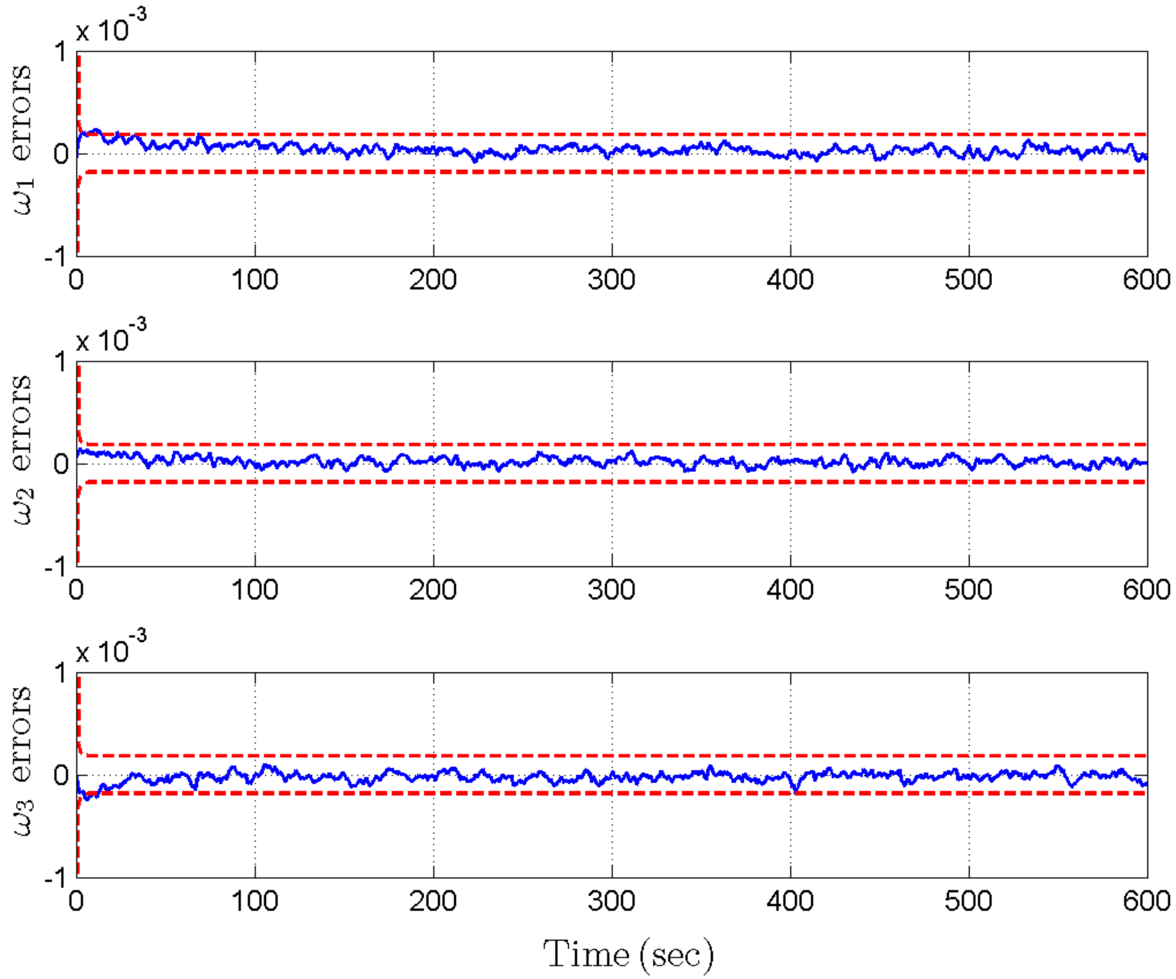
Numerical Example (1/6)

Simulation Parameters

Parameter	Symbol	Value	Unit
Moment of inertia	J	diag [14.2, 17.3, 20.3]	kg·m^2
Sensor noise	ν	1e-4	rad/s
Torque	u	[1 1 -2]^T * 1e-3	Nm
Initial MOI	J_i	0.5*J	kg·m^2

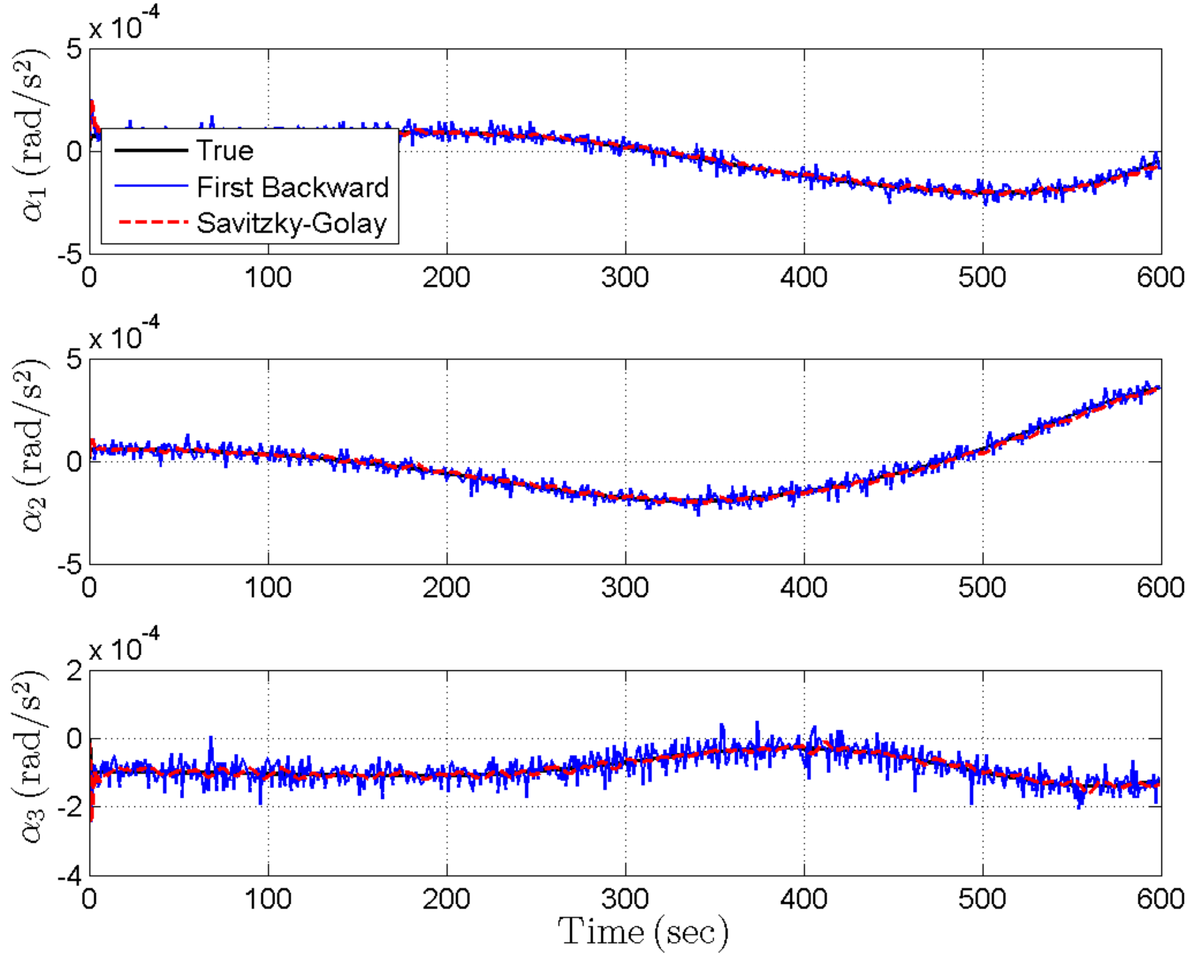
Numerical Example (2/6)

Angular velocity filtering results using the EKF



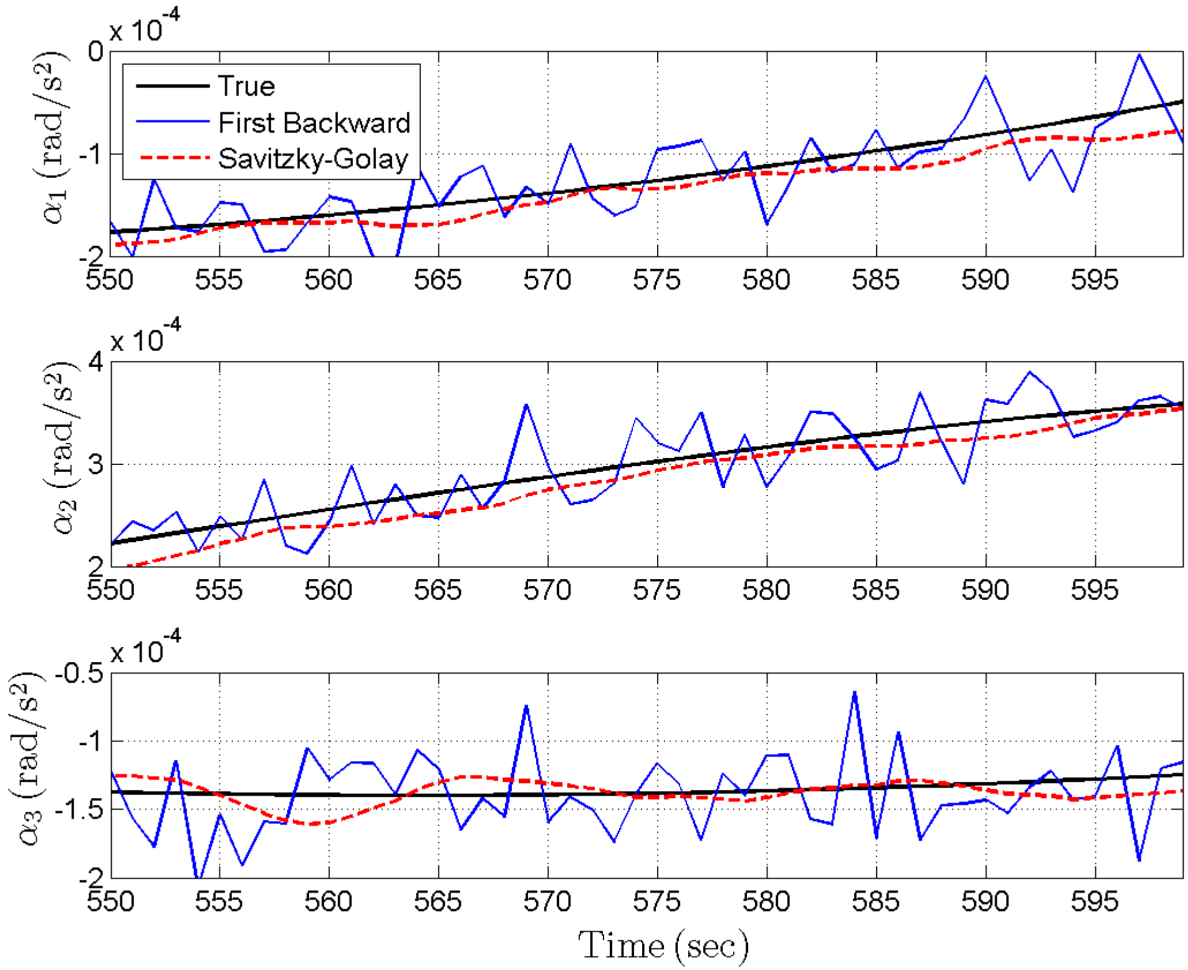
Numerical Example (3/6)

Angular acceleration calculation results using the SGF



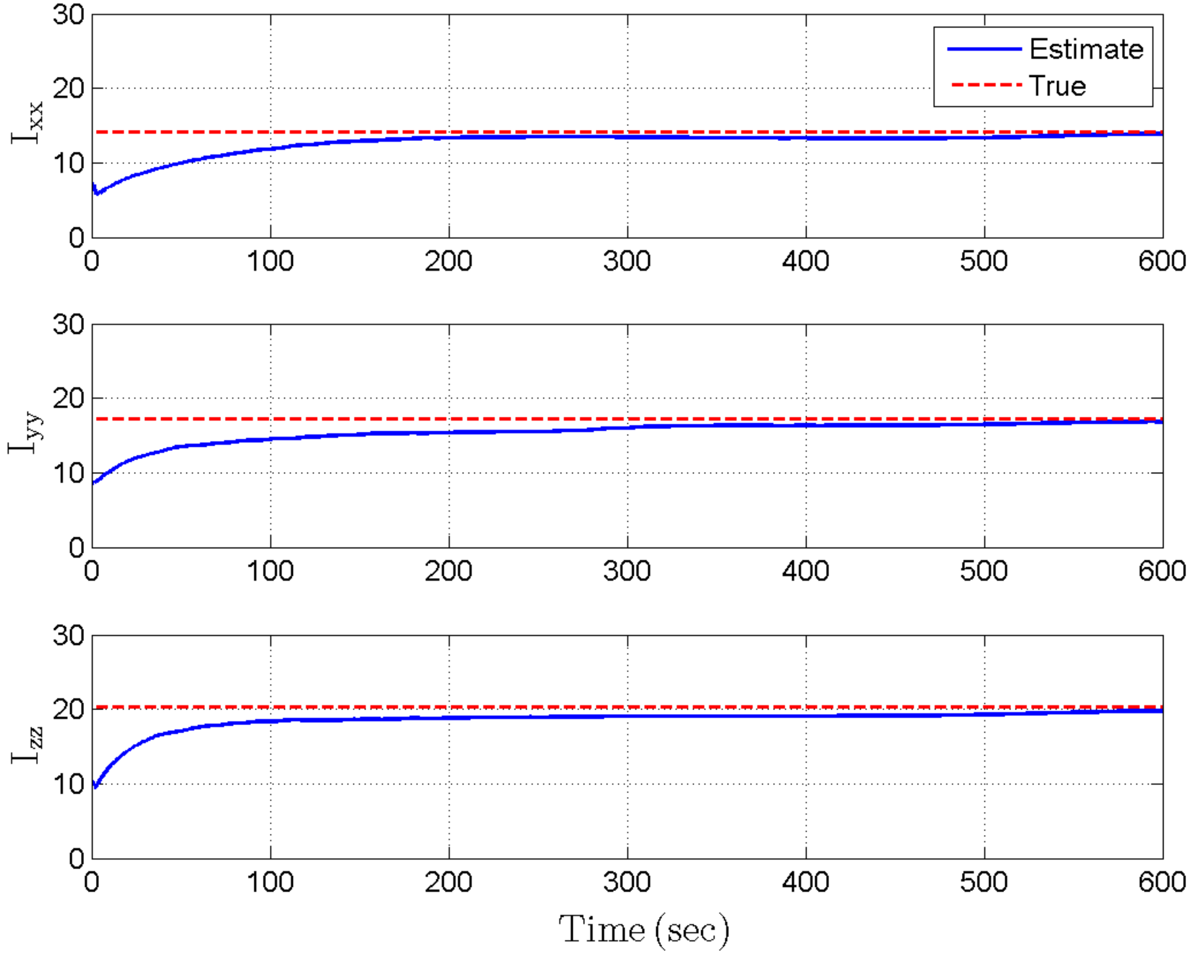
Numerical Example (4/6)

Angular acceleration calculation results using the SGF



Numerical Example (5/6)

MOI estimation results using the RLS



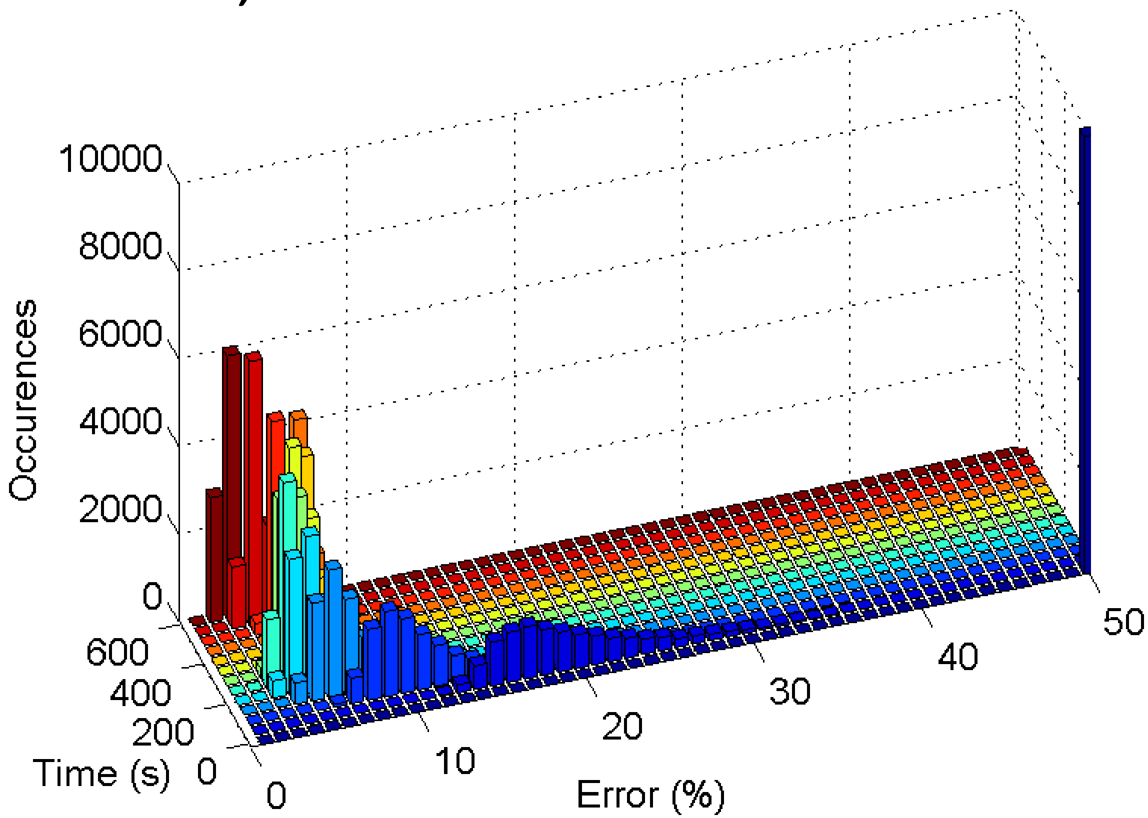
Error (%)

	Average	Final
J_{xx}	10.71	2.38
J_{yy}	10.66	2.60
J_{zz}	8.47	2.50

Numerical Example (6/6)

Monte Carlo test for validating results

- 10,000 trials



Average Error (%)

Time (sec)	J_{xx}	J_{yy}	J_{zz}
0	50	50	50
100	10.50	11.57	8.30
200	5.05	6.64	6.43
300	4.78	5.56	5.43
400	5.84	5.16	5.32
500	5.60	4.63	4.76
600	2.84	2.89	2.84

Conclusion

- MOI estimation is successfully performed in real-time using RLS.
  - EKF for angular velocity / SGF for angular acceleration
- EKF and SGF works for filtering the data but the estimation performances depends on the window size (tuning parameter).
  - Window size ↑ ⇒ SGF performance ↑
    ⇒ time lag ↑
  - **Trade-off is required!!**

Future Works

- Determination for the best value of window size for the best estimates (MOI)
- Application for varying MOI
  - based on the modified equation which includes time derivative of the MOI

$$\dot{\boldsymbol{\omega}} = \boldsymbol{f}\,(J,\boldsymbol{\omega}(t),\mathbf{u}(t),t) \rightarrow \dot{\boldsymbol{\omega}} = \boldsymbol{f}\,(J(t),\dot{J}(t),\boldsymbol{\omega}(t),\mathbf{u}(t),t)$$

- Case study for varying control torque
  - To avoid arbitrary control torque generation irrelevant with the mission
- Extension for off-diagonal element of the MOI estimation

Questions & Answers