Astrodynamics

The "Horseshoe" Orbit of Near-Earth Object 2013 BS_{45}

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Figure 1. The 1.8 m Spacewatch telescope is pictured inside its protective dome at Kitt Peak, Arizona (photograph by Robert S. McMillan).

1. Earth-Based Discovery

Discovered by the Spacewatch 1.8 m telescope (see Figure 1) on 20 January 2013, near-Earth object (NEO) 2013 BS_{45} closely encountered Earth at a range of 0.0126 AU (4.9 lunar distances or 1.88 million km) on 12 February 2013.

As is typical among NEO discoveries made in the night sky prior to closest Earth approach with observations of our planet's night sky, 2013 BS_{45} crosses Earth's orbit inbound towards the Sun. It reaches perihelion on 29 April 2013 at 0.92 AU or 92% of Earth's mean distance from the Sun. Figure 2 is a plot of 2013 BS_{45}, Earth, and Mars as they orbit the Sun during 2013.

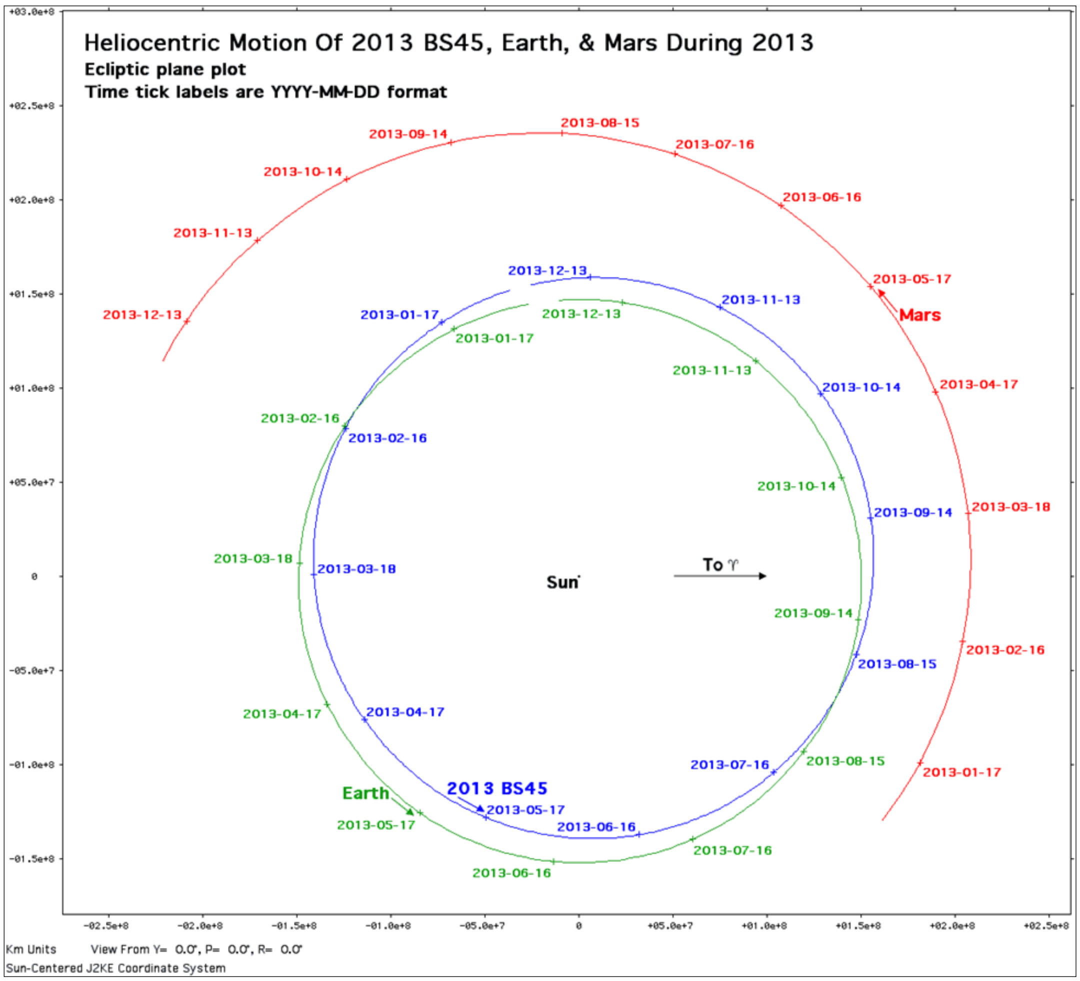

Figure 2. Orbits of 2013 BS_{45} (blue), Earth (green), and Mars (red) are plotted during year 2013 in a non-rotating (inertial) Sun-centered (heliocentric) coordinate system. The plot plane coincides with that of Earth's orbit, the ecliptic, and 2013 BS_{45}'s orbit is inclined to the ecliptic by less than 1°.

Before moving into Earth's daytime sky circa 9 February 2013, about 80 optical observations were being processed by the Jet Propulsion Laboratory (JPL) to produce 2013 BS_{45} ephemerides with maximum position uncertainties equivalent to hundreds of minutes in heliocentric motion a century in the past or future. During mid-February, planetary radar observations conducted at Goldstone, CA had reduced this uncertainty to the order of 10 minutes.

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2. Accessibility for Human Spaceflight

Since the orbits of Earth and 2013 BS_{45} are so similar, this NEO should be highly accessible for human spaceflight (HSF) whenever it closely approaches Earth. Close approaches are a necessary condition for HSF accessibility because of the trade between speed and distance in this context. Unless distance between Earth and NEO is small, spacecraft speed must be increased to impractical levels in order to complete a roundtrip within the time limits of human exposure to confinement, galactic cosmic radiation, and microgravity. This trade, along with many others, is made by NEO HSF Accessible Targets Study (NHATS, pronounced "gnats") software. The Goddard Space Flight Center, in cooperation with JPL, generates and posts NHATS data to http://neo.jpl.nasa.gov/nhats/ on a daily basis.

Three-dimensional pork chop charts (PCCs) succinctly summarize mission viability under NHATS criteria. In a PCC plotted by NHATS software, the horizontal axis is Earth departure date, and the vertical axis is roundtrip flight time in days. Each pixel in a PCC's domain is colored according to total mission change-in-velocity Δv_{TOT} in km/s[1]. White pixels violate one or more NHATS mission viability criteria. Excessive Δv_{TOT} or mission duration will generally result from attempting to cover an excessive roundtrip distance. The PCC for 2013 BS_{45} appears in Figure 3.

[1] In NHATS software, Δv_{TOT} is computed as the sum of impulses required to depart a circular Earth orbit at 400 km height targeting NEO intercept, achieve NEO rendezvous, perform NEO departure targeting Earth return, and ensure Earth's atmosphere is entered at a speed of 12.0 km/s if this value would otherwise be exceeded.

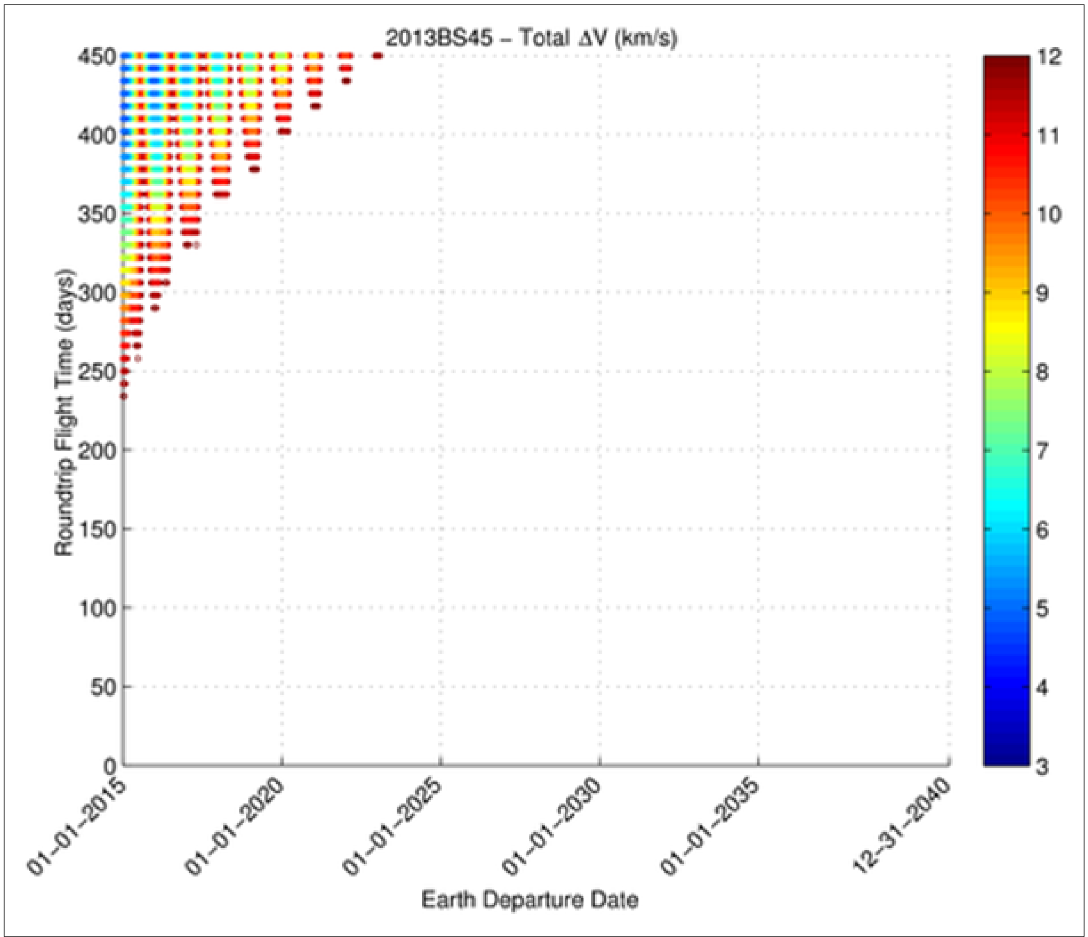

Figure 3. The 2013 BS_{45} PCC posted 20 February 2013 at the NHATS website shows viable mission opportunities on the wane at the earliest NHATS-compliant Earth departure dates during year 2015. In accord with the Δv_{TOT} color legend at right, a NEO with ideal HSF

If NHATS mission viability criteria included Earth departure dates circa year 2013, a PCC for 2013 BS_{45} would be filled with deep blue pixels at relatively short roundtrip flight times for those dates. Unfortunately, about half of these nearly ideal HSF mission opportunities would have been history by the time 2013 BS_{45} was discovered. As matters stand in early 2013, only about 3 or 4 years would remain to plan, assemble, and depart Earth before a HSF mission to 2013 BS_{45} became impractical due to excessive duration and/or excessive Δv_{TOT}.

3. Horseshoe Motion With Respect To The Sun-Earth Line

How long will it be before 2013 BS_{45} again makes close approaches to Earth and NHATS-viable mission opportunities resume? Although

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this NEO's orbit requires some refinement before long-duration predictions can be made with high confidence, the answer appears to be "about 80 years". This interval between successive clusters of close Earth approaches is equivalent to a NEO's *synodic period*. Figure 4 plots motion of 2013 BS_{45} in the ecliptic plane with respect to a rotating Sun-centered coordinate system in which the Sun-Earth line is fixed. This plot accounts for all manner of perturbations to 2013 BS_{45}'s heliocentric motion, chief among these being Earth's gravity. Figure 4 motion spans one synodic period, extending from year 1932, when 2013 BS_{45} last began a series of close Earth approaches, through year 2015, when the current series of close Earth approaches ends.

Kepler's third law is often used to compute the heliocentric angular rate ω of a NEO's orbit using the following formula, in which a is the NEO orbit's heliocentric semi-major axis and μ is the Sun's reduced mass.

$$\omega = \sqrt{\frac{\mu}{a^3}}$$

With Earth's heliocentric angular rate ω_E well determined, the NEO's synodic period T_S is the time required for the difference in angular rate between NEO and Earth to ac-

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[2] In this context, "Earth's vicinity" refers to observations made at Earth's surface and at contemplated space-based locations ranging out to Sun-Earth libration points about 1.5 million km from Earth along the Sun-Earth line. These libration points are commonly referred to as SEL1 (lying between Earth and the Sun) and SEL2 (lying beyond Earth from the Sun).

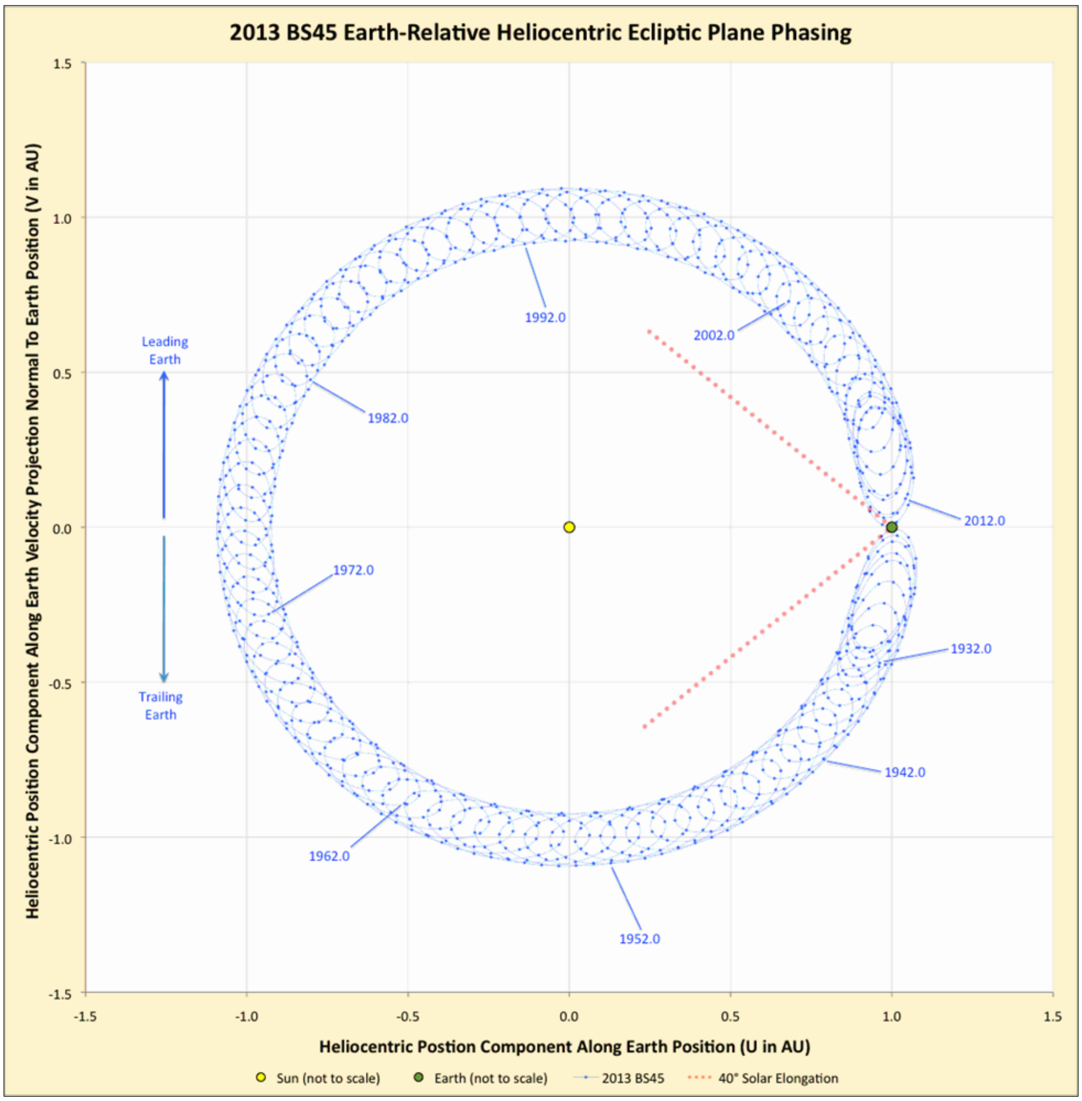

Figure 4. Heliocentric motion of 2013 BS_{45}, beginning with close Earth approaches in year 1932 and ending with other close approaches in year 2015, is plotted in the ecliptic plane with respect to a fixed Sun-Earth line.

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cumulate a full revolution. Therefore, $T_S = 2\pi / (\omega - \omega_E)$.

All NEO orbits crossing that of Earth are grouped into two families. Those Earth-crossers with $\omega < \omega_E$ are assigned to the Apollo family, and those with $\omega > \omega_E$ are assigned to the Aten family. A sign convention is embedded in the T_S formula whereby Aten family orbits produce a positive value, and Apollo family orbits produce a negative value. Table 1 presents examples of T_S computations for 2013 BS_{45} during early 2013 as it undergoes an Apollo-to-Aten transition.

Table 1. When Kepler's third law is used to compute tabulated 2013 BS_{45} synodic period T_S values in early 2013, wildly varying results are obtained as the NEO's heliocentric angular rate transitions from less than Earth's to greater than Earth's. If Earth gravity perturbations are included when modeling 2013 BS_{45}'s orbit, a T_S value near 80 years is inferred.

2013 UT	T_S (years)
05.0 Jan	-201.283
15.0 Jan	-209.741
25.0 Jan	-226.579
04.0 Feb	-283.056
14.0 Feb	-3873.767
24.0 Feb	+477.737
06.0 Mar	+355.733
16.0 Mar	+316.713
26.0 Mar	+297.532
05.0 Apr	+286.492

From Table 1's example, it is evident T_S computations ignoring Earth gravity perturbations on a heliocentric NEO orbit cannot produce consistent or meaningful results at times when those perturbations are significant. In such instances, a thorough analysis of the perturbed orbit must be conducted from one set of close Earth approaches through the next set to infer the actual T_S.

As annotated in Figure 4, the plot's vertical "V" coordinate signifies whether 2013 BS_{45} leads (positive V) or trails (negative V) Earth as they orbit the Sun. Position of 2013 BS_{45} in Figure 4 is annotated for the new year at 10-year intervals, beginning with the initial point at "1932.0". Proceeding chronologically from this initial point, 2013 BS_{45} trails Earth until the mid -1970s when it lies across the solar system from our planet and is highly inaccessible for HSF. Thereafter, 2013 BS_{45} grows progressively closer to Earth from positions leading it in orbit about the Sun.

The dotted red "v" whose apex coincides with Earth in Figure 4 denotes a *solar exclusion zone* (SEZ) in which a NEO cannot be observed from Earth's vicinity because its apparent solar elongation is less than 40°[2]. Although this zone has infinite extent along Figure 4's -U axis, its boundary is only drawn out to a geocentric range of 1 AU in Figure 4 because NEOs are typically so small and intrinsically faint they cannot be detected very far from Earth. Indeed, 2013 BS_{45} was discovered only after it had closed within 0.044 AU or 6.6 million km from Earth. From its apparent brightness and known distance from Earth, NHATS software estimates 2013 BS_{45} is 12 to 53 m in diameter assuming a reflectivity range spanning most NEOs of known size. The SEZ rules out observing an appreciable percentage of close Earth approach points in Figure 4. A small NEO with shorter T_S than 2013 BS_{45}'s might have flown past Earth too quickly and evaded discovery during the brief interval it was close enough to observe.

A close examination of yearly "loops" made by 2013 BS_{45} in Figure 4 shows they tend to bunch-up when nearest to Earth. This is the graphic manifestation of variations in ω previously noted and arises from Earth gravity perturbations to 2013 BS_{45}'s heliocentric orbit. Circa year 1932, when 2013 BS_{45} closely trails Earth, these perturbations decrease ω from slightly more than ω_E to slightly less than ω_E. In terms of NEO orbit families, 2013 BS_{45} transitions from an Aten to an Apollo and is never able to overtake Earth. During year 2013, similar Earth perturbations are at work to increase 2013 BS_{45}'s ω just before Earth would otherwise overtake it. In this scenario, 2013 BS_{45} is transformed from an Apollo back to an Aten. Because of the gap surrounding Earth in Figure 4, 2013 BS_{45}

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is said to be in a "horseshoe" orbit. Figure 5 illustrates 2013 BS_{45}'s Apollo-to-Aten transition in detail from year 2011 into year 2016.

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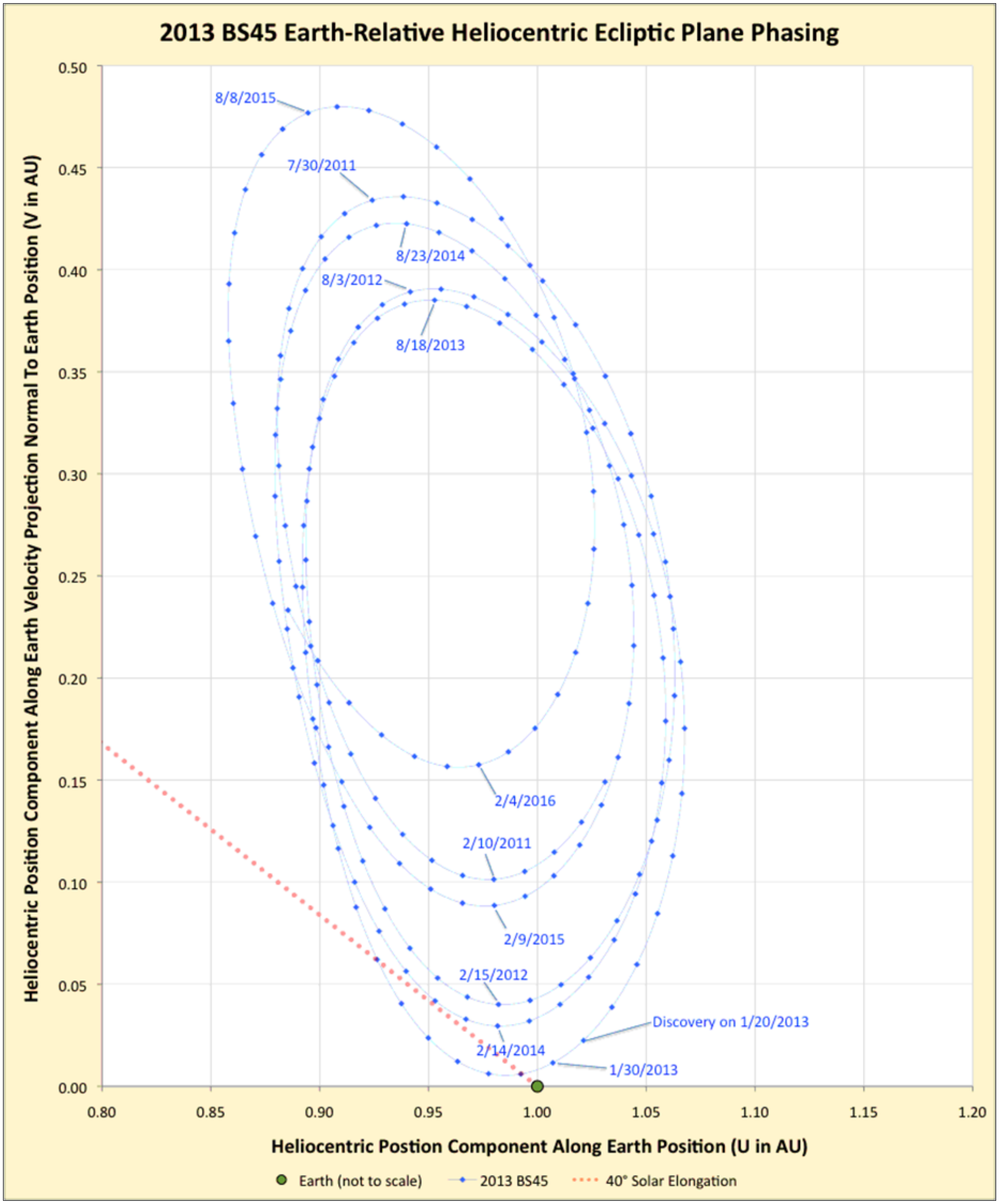

Figure 5. The "horseshoe" orbit turnaround is plotted with respect to the Sun-Earth line as 2013 BS_{45} transitions from the Apollo to the Aten orbit family during years 2011 to 2016. This is a highly magnified segment of Figure 4's domain.

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Trajectory markers in the Figure 5 plot are placed at 10-day intervals. Along the trajectory arc passing closest to Earth at the bottom of this plot, note only 20 days following 2013 BS_{45} discovery are available in which to observe this NEO from Earth before it drifts into the SEZ. By the time 2013 BS_{45} departs the SEZ in early April 2013, 70 days after discovery, it is likely too far from Earth to detect. It will hopefully be recovered during its next close approach during early 2014, when uncertainties in its orbit could then be appreciably reduced.

It is also useful to consult Figure 5 with HSF accessibility in mind. A necessary condition among all NEO round-trip mission designs boasting relatively short duration and low Δv_{TOT} is Earth-NEO distance less than 0.1 AU at some point during the mission timeline. This criterion applied to Figure 5 corroborates the best mission opportunities are ending in 2015 just as the Figure 3 PCC's time domain opens under NHATS criteria.

4. Discovering NEOs Well In Advance Of Their Close Earth Approaches

The NEOCam concept presented by JPL/Dr. Amy Mainzer in 2009 (ref. link) proposes a NEO survey conducted from SEL1 at solar elongations from 40° to 125°. Assume this instrument is capable of detecting objects as faint as apparent visual magnitude m = +24. With this assumed sensitivity and deployment sufficiently far in the past, NEOCam would have discovered 2013 BS_{45} as early as January 2011 at a solar elongation near 106° and geocentric distance near 0.14 AU. It should be noted that NEOCam is designed to observe infrared emissions quite distinct from reflected visible light simulations leading to this January 2011 estimate. Nevertheless, a 2013 BS_{45} discovery two years before the actual event could have allowed sufficient time to mount a viable HSF mission. A "launch on need" capability, placing the mission in a high state of readiness before its destination is known, might be necessary to visit serendipitously discovered NEOs offering mission opportunities whose Earth departure dates are imminent.

As a means to observe NEOs in the SEZ and all around Earth's orbit during reasonable time intervals, consider a NEO survey telescope operating in interplanetary space with perihelion at 0.700 AU (near the distance of Venus from the Sun) and aphelion at 0.882 AU (near 2013 BS_{45}'s perihelion distance). Such an instrument would have a T_S of only 2.37 years. Because it remains well inside Earth's orbit, nearby NEOs observed by the telescope in proximity to Earth's orbit always have solar elongations greater than 90°. As such, each observation tends to be of a well-illuminated NEO surface near its maximum possible brightness from a given distance.

Figure 6 plots motion of this hypothetical telescope for 10 years using a coordinate system identical to that of Figure 4. This plot begins with the telescope arbitrarily at aphelion near the +U axis on the date 2013 BS_{45} was discovered. It then extends 10 years into the future. A point near each telescope aphelion is annotated with the corresponding date in Figure 6 as 4.2 synodic periods convolve around Earth's orbit. A telescope in this orbit with m = +24 sensitivity could detect a NEO like 2013 BS_{45} years or decades before viable HSF mission opportunities would arise.

5. Summary

The orbit of 2013 BS_{45} serves as a specific example supporting four important precepts associated with NEOs of high HSF accessibility. First, the most accessible NEO orbits tend to be the most Earthlike.

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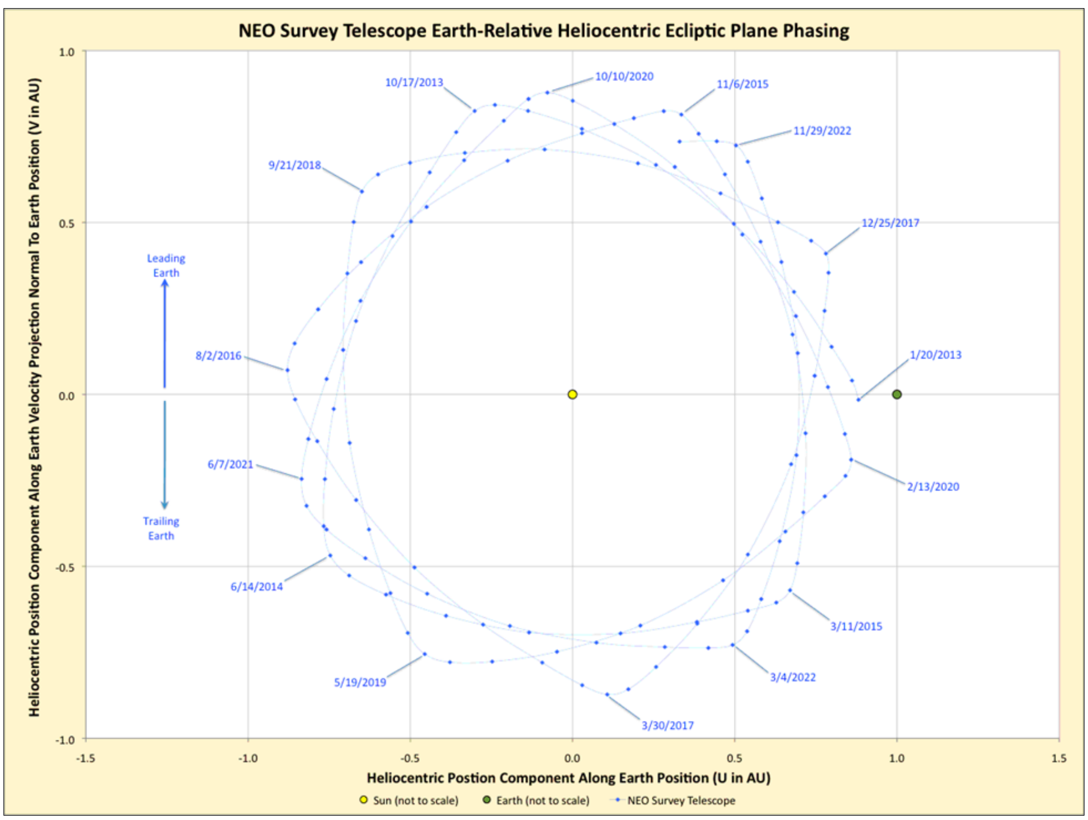

Figure 6. Motion of a notional NEO survey telescope operating between the orbits of Venus and Earth is plotted with respect to a fixed Sun-Earth line in the ecliptic plane. The 10-year interval of this plot spans 4.2 synodic periods for the telescope, ensuring all sectors of interplanetary space near Earth's orbit are observed on multiple occasions.

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Such orbits have protracted HSF launch opportunities several years in duration, but these accessibility seasons may be separated by intervals from decades to a century.

Second, close NEO approaches to Earth associated with HSF mission opportunities are also the only occasions permitting Earthbound observers to detect small ones ~100 m in diameter or less. This leaves little time to prepare and dispatch a HSF mission during an accessibility season.

Third, by conducting a NEO survey from the SEL1 libration point or from interplanetary space between the orbits of Venus and Earth, a potential HSF destination such as 2013 BS_{45} can be observed years or decades in advance of a close Earth approach. These observations will likely leave adequate time to prepare for and utilize the most practical HSF mission opportunities.

Fourth, some Earthlike NEO orbits display a horseshoe character in which close approaches leading and trailing Earth are achieved with regularity, but the Sun-Earth line is never crossed. Earth gravity perturbations during these close approaches impart turnarounds in the heliocentric rate at which the NEO is chasing Earth or vice-versa. Because NEOs in horseshoe orbits possess extremely long synodic periods and have only been observed for the past decade or two, little is certain about the long-term dynamical stability of such orbits.

Acknowledgments

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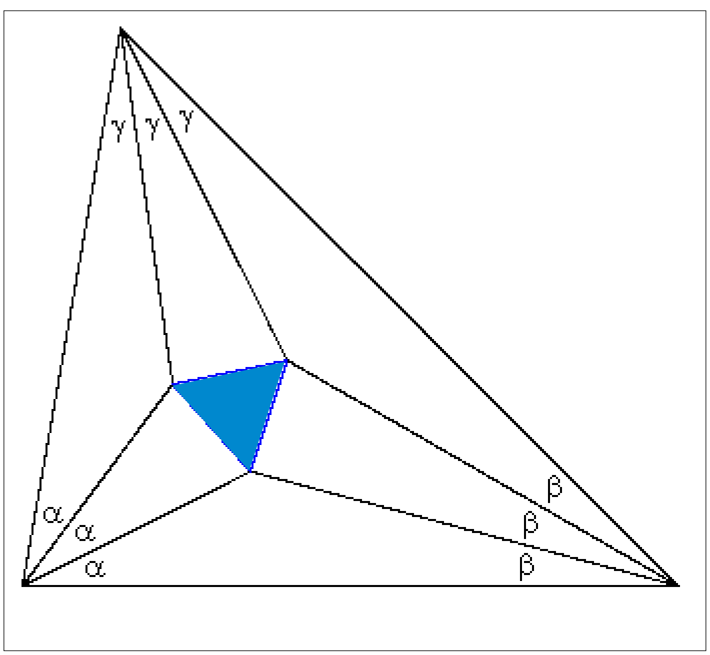
Cranium Cruncher

Prove this Triangle is Equilateral

DOUGLAS YAZELL, EDITOR, FILLING IN FOR DR. STEVEN E. EVERETT



Above: A web site called MathPages by Kevin Brown (author) includes Morley's trisection theorem. Image credit: Kevin Brown.

My great friend Jean-Marie Lemaitre showed me this brain teaser about two years ago. He showed me a quick sketch like the figure at left. On May 29, 2013, I was able to find the question and answer thanks to a Google search which found an unusual web site. The web site, MathPages, seems to be the creation of author Kevin Brown. The question and answer appear here on his web site, along with excellent history notes. By the way, Jean-Marie teaches mathematics in Hong Kong at the moment. He is no relation to Georges Lemaitre. The fifth Automated Transfer Vehicle (ATV) from the European Space Agency is named after Georges Lemaitre. Based in part on that excellent ATV program success, NASA's first Orion crew capsule spacecraft will have an ESA Service Module.

MathPages includes animated GIFs. My iPad 1 news aggregator application Flipboard showed me a web site with excellent animated GIFs created by PATAKK. Below is a screen capture image of one of those animated GIFs.