**Exploration with Quantum Computation: Mars Drone**

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**Questions, Answers, and Suggestions are welcome**

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**Quantum Information and Computation**

A new quantum theory of communication and computation is emerging, in which the stuff transmitted or processed is not classical information, but arbitrary superpositions of quantum states.

Charles H. Bennett

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**Abstract**

We investigate the conditions for entanglement for a system of two atoms and two photon modes in vacuum, using the Jaynes-Cummings model in the rotating-wave approximation. It is found that the existing results, that the the strength of entanglement is a periodic function of time. We explicitly show that one gets results

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**Entanglement in a Jaynes-Cummings Model with Two Atoms and Two Photon Modes**

Samina S. Masood, Allen Miller

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**A von Neumann Entropy Measure of Entanglement Transfer in a Double Jaynes-Cummings Model**

Samina S. Masood, Allen Miller

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**Two-Dimensional Quantum Search Algorithm**

Arti Changuj and Samina Masood

Department of Physics, University of Houston Clear Lake

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**Topological Quantum Computation**

Sankar Das Sarma, Michael Freedman, and Chetan Nayak

The search for a large-scale, error-free quantum computer is reaching an intellectual juncture at which semiconductor physics, knot theory, string theory, anyons, and quantum Hall effects are all coming together to produce quantum immunity.

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**Questions, Answers, and Suggestions are welcome**

**Physics Today Articles, Dr. Masood Articles, AIAA Aerospace America Articles, Zettili Textbook**
Quantum computation is envisioned as the evolution of a system from an initial state to a final state by a unitary transform operator varying with time and the states are the form of a qbit. Anything able to be expressed as at least two states in the form of a qbit may be used as the hardware for quantum computation. Examples include photon phase, photon intensity, two atomic states, spin-half particles which are fermions entailing quarks and leptons (e.g. electrons and neutrinos), photon polarization, and an existing D-Wave Computer with electrons of Niobium at temperatures approximating absolute zero. The potential of quantum computation is in the application of quantum properties of superposition, interference, entanglement, and uncertainty. Problems of decoherence and absence of data compression may be addressed by topology of 2-D non-abelian anyons and non-orthogonal states, respectively. There is success of verification and validation of flight software by Lockheed Martin and Boeing running D-Wave. Uncertainty is appropriate for Mars drones with quantum artificial intelligence as is entanglement of 2 atoms and 2 photon modes for communication, expandable to n-particle states. Machine learning of robotics will improve by verification and validation backtesting of quantum algorithms with (1) increasing available data through the eMerge app concept to enable public participation and (2) using the quantum search in 2-D. Like the acceptance of new and existing physics equations and the eMerge app, Ease of use and Usefulness lead to quantum computation acceptance and associated acceleration.
Research Structure

Introduction
- Quantum computation is envisioned as the evolution of a system from an initial state to a final state by a unitary transform operator varying with time and the states are the form of a qbit.
- Anything able to be expressed as at least two states in the form of a qbit may be used as the hardware for quantum computation.

Theoretical background
- Examples include photon phase, photon intensity, two atomic states, spin-half particles which are fermions entailing quarks and leptons (e.g. electrons and neutrinos), photon polarization, and an existing D-Wave Computer with electrons of Niobium at temperatures approximating absolute zero.
- The potential of quantum computation is in the application of quantum properties of superposition, interference, entanglement, and uncertainty.
- Problems of decoherence and absence of data compression may be addressed by topology of 2-D non-abelian anyons and non-orthogonal states, respectively.

Detailed exploration of the topic with pictures and formulas
- There is success of verification and validation of flight software by Lockheed Martin and Boeing running D-Wave.
- Uncertainty is appropriate for Mars drones with quantum artificial intelligence as is entanglement of 2 atoms and 2 photon modes for communication, expandable to $n$-particle states.
- Machine learning of robotics will improve by verification and validation backtesting of quantum algorithms with (1) increasing available data through the eMerge app concept to enable public participation and (2) using the quantum search in 2-D.

Brief summary
- Like the acceptance of new and existing physics equations and the eMerge app, Ease of use and Usefulness lead to quantum computation acceptance and associated acceleration.
Quantum Computation

Introduction

• Quantum computation is envisioned as the evolution of a system from an initial state to a final state by a unitary transform operator varying with time and the states are the form of a qbit.

• Anything able to be expressed as at least two states in the form of a qbit may be used as the hardware for quantum computation.

$$|\psi\rangle = c_0 |0\rangle + c_1 |1\rangle$$
Currently, quantum computation is envisioned as the evolution of a system from an initial state to a final state by a unitary transform operator varying with time and the states are the form of a qbit.

Quantum computation is currently envisioned as the evolution of a system. The purpose of creating quantum computation is to enable fast and secure computation. Understanding quantum mechanics is essential to understanding quantum computation because quantum mechanics explains the fundamental operations at the smallest scale. Applying quantum mechanics to computations has promise to operate at the smallest scale. Energy is conveyed in discrete of units called quanta. Objects can be a superposition of different states simultaneously. Some calculations could be performed fast because of superposition. For example, lasers may be used to switch a calcium ion between two states. Quantum computation has the potential to revolutionize the economy, research, and physics from the information age to quantum age like the information age revolutionized the industrial age (Many sources for example Hadhazy, pp. 22-27, 2016; Worden, p. 16, 2015).
Introduction

The purpose of this literature review is to review relevant research and theory, which for this paper is the two Physics Today articles, the three research papers by Dr. Masood, and the Zettili textbook, to formulate the problem as the primary research question. The problem is to apply quantum mechanics to computation: quantum computation. So, the purpose of this paper is to describe and demonstrate quantum mechanics’ application to quantum computation.

A literature review focusing on Physics Today articles indicates quantum computation is envisioned as the evolution of a system from an initial state. Specifically, the initial state is in the form of a wave function $\psi$ with a zero subscript, where alternative sources usually uses un-prime as the initial state

$$|\psi_0\rangle \text{ or } |\psi\rangle$$

As time ($t$) continues, evolution occurs by a unitary operator $U$ transforming the initial state

$$\tilde{U} = f(t)$$

such that the final state is in the form of a wave function $\psi$ with a 1 subscript, where the textbook usually uses prime as the final state

$$|\psi_1\rangle = \tilde{U}(t)|\psi_0\rangle \text{ or } |\psi'\rangle = \tilde{U}(t)|\psi\rangle$$

in which the states are of the form of a qbit

$$|\psi\rangle = c_0|0\rangle + c_1|1\rangle$$

where $c$ is complex. The general form for $n$ qbits is untangled

$$\psi = \sum_{x=00\ldots0}^{11\ldots1} c_x |x\rangle$$

$$= c_0|0\rangle + c_1|1\rangle \quad \text{ for } n = 1$$

$$= c_{00}|00\rangle + c_{10}|10\rangle + c_{01}|01\rangle + c_{11}|11\rangle \quad \text{ for } n = 2$$

$$= c_{00\ldots0}|00\ldots0\rangle + c_{10\ldots0}|10\ldots0\rangle + \ldots + c_{11\ldots1}|11\ldots1\rangle \quad \text{ for } n$$

$$= c_{00\ldots0}|00\ldots0\rangle + \ldots + c_{11\ldots1}|11\ldots1\rangle \quad \text{ for } n$$

Untangling the general form for $n$ qbits
Introduction

- The transformation consists of an operator and a wave function. Either one or the other or both may be a function of time \(t\). The unitary transformation operator may be a function of time known as the Heisenberg picture, or the wave function may be a function of time known as the Schrödinger picture, or both the may be a function of time known as the Dirac picture, also known as the Interaction picture. For a unitary transform operator \(\hat{U}(t)\) varying with time, the Heisenberg picture is appropriate. The time dependence of the state is frozen. The Dirac picture may be appropriate if the wave function is also a function of time. In either case, the nature of the unitary transform operator \(\hat{U}(t)\) is worth reviewing (Zettili, 2009, p. 572).

- The time evolution of a state \(|\psi(t)\rangle\) can be expressed by means of the propagator, or time-evolution operator, \(\hat{U}(t, t_0)\), as follows:

\[
|\psi(t)\rangle = \hat{U}(t, t_0)|\psi_0(t)\rangle
\]

With

\[
\hat{U}(t, t_0) = e^{-i(t-t_0)\hat{H}/\hbar}
\]

- The operator \(\hat{U}(t, t_0)\) is unitary,

\[
\hat{U}^\dagger(t, t_0)\hat{U}(t, t_0) = I
\]

- and satisfies these properties as delineated in the textbook (Zettili, 2009, p. 572)

\[
\hat{U}(t, t_0) = I
\]

\[
\hat{U}^\dagger(t, t_0) = \hat{U}^{-1}(t, t_0) = \hat{U}(t_0, t)
\]

\[
\hat{U}(t_1, t_2)\hat{U}(t_2, t_3) = \hat{U}(t_1, t_3)
\]

- As a system absorbs or emits radiation, it undergoes transitions from one state to another (Zettili, 2009, p. 571, §10.1). States may be represented as wave functions, transitions may be represented by operators, and there are many representations of wave functions and operators in quantum mechanics. The connection between the various representations is provided by unitary transformations (Zettili, 2009, p. 571, §10.2). The three pictures encountered most frequently in quantum mechanics are the Schrödinger picture, the Heisenberg picture, and the Interaction picture. The Schrödinger picture is useful when describing phenomena with time-independent Hamiltonians, whereas the Interaction and Heisenberg pictures are useful when describing phenomena with time-dependent Hamiltonians. (Zettili, 2009, p. 571, §10.2)

Dirac picture: \(|\psi(t)\rangle = \hat{U}(t, t_0)|\psi_0(t)\rangle\)
Introduction

- As an example of unitary transformation operator, all rotation operators are unitary (Zettili, 2009, Chap 7, (7.41), p. 396). Specifically Exercises 2.16, p. 157, 2.40 & 2.41, p. 161 show:
  
  \[
  \begin{pmatrix}
  \cos \theta & \sin \theta & 0 \\
  -\sin \theta & \cos \theta & 0 \\
  0 & 0 & 1
  \end{pmatrix},
  \begin{pmatrix}
  1 & -1 & 0 \\
  0 & 0 & 1 \\
  1 & 0 & 0
  \end{pmatrix} 90^\circ \text{counter clockwise},
  \begin{pmatrix}
  0 & 0 & -1 \\
  0 & 1 & 0 \\
  1 & 0 & 0
  \end{pmatrix} 90^\circ \text{clockwise}
  \]

- An advantage of unitary operators is the original state is returned by operating with the adjoint:
  
  for \( \psi' = U\psi \) then operating with the adjoint:
  
  \[ U^\dagger \psi' = U^\dagger U\psi = U^{-1}U\psi = \psi \]

- Another example is the transformation matrix formed by Clebsch-Gordan coefficients, which is also unitary (Zettili, 2009, Chap 7, Example 7.3 pp. 411-415). Specifically, Clebsch-Gordan coefficients are matrix elements of a unitary transform (Zettili, 2009, Chap 7, (7.122), p. 406). An example unitary transformation depending on time is worth considering where \( E_n \) are the energy eigenvalues and \( |\psi_1\rangle \) are the eigenfunctions of the Hamiltonian \( \hat{H} \) only if \( |\psi_1\rangle = |\psi_0\rangle \):
  
  \[ \hat{U}(t) = e^{-i(t-t_0)\hat{H}/\hbar} \] so \( |\psi_1\rangle = e^{-i(t-t_0)\hat{H}/\hbar}|\psi_0\rangle \) or \( |\psi_1\rangle = e^{-i(t-t_0)E_n/\hbar}|\psi_0\rangle \)

- More generally, where \( \Omega \) or \( A \) an operator with \( \omega_n \) or \( a_n \) as the eigenvalues, the unitary transformation is:
  
  \[ \hat{U}(t) = e^{-i\hat{A}/\hbar} \text{ or } U(t) = e^{-it\hat{A}/\hbar} \]

- If “Quantum data processing consists of applying a sequence of unitary transformations to the state vector \( \Psi \)” (Bennett, 1995, p. 24):
  
  \[ \hat{U}_N \text{ where } \hat{U}_N \text{ operates on } N \text{ number of unknown qbits (p. 26)}, \]

then the sequence of unitary transformations may be a series of unitary transformations that do not depend on time. Therefore the unitary transformations may not be time dependent. In addition, the sequence may not be time dependent. But according to Das Sarma, Freedman, and Nayak (2006, p. 32), the unitary transform may be any unitary transform operator applicable and needed for each situation and is a function of time:

- \( U(t) \) where \( U = f(t) \)

Quantum mechanics should be successfully applicable to computation (successfully meaning small, fast, encrypted, and economical), using unitary transform of states, resulting in successful quantum computation.
Introduction

• Anything able to be expressed as at least two states in the form of a qbit may be used as the hardware for quantum computation.

• Are we limiting computation to an old paradigm? The research of quantum computation starts with many questions and questions remain unanswered. Assuming computation is limited to forms of states expressed with at least ones and zeros may be too limiting. Maybe there is another way?

• Spin orientations up and down of electrons or nuclei are suitable qbits, as are atoms which can be excited to a higher energy states. The literature uses many formulations for the qbit. This paper considers converting alternative forms of the qbit to a common formulation of the form 1’s and 0’s. Specifically, consideration is given to using the same qbit formulation throughout the paper by replacing spin-up $|\frac{1}{2}, \frac{1}{2}\rangle$, spin-down $|\frac{1}{2}, -\frac{1}{2}\rangle$ and nuclear spin up $|0\rangle$ and down $|1\rangle$, and ground state $|0; \downarrow\rangle_A$ or $|G_A, a\rangle$ excited state: $0; |+\rangle_B$ or $|E_A, a\rangle$ where $|a\rangle$ are superposed spin states (Chamoli & Masood, 2010, p. 4) or ground state $|\downarrow\rangle$ or excited state $|\uparrow\rangle$ (Masood and Allen Miller, 2014, p.1) with the same qbit equation

  $|\psi\rangle = c_0 |0\rangle + c_1 |1\rangle$

• Parametrization and compactification are approaches to quantum mechanics problems and are applicable to quantum computation. A qbit as 1’s and 0’s for spin-up and spin-down particles may be considered parametrization. The reader naturally links the connection to 1’s and 0’s when given two clearly independent options such as spin-up and spin-down or ground state and excited state. Spin-up as $|\frac{1}{2}, \frac{1}{2}\rangle$ compared to 1’s and 0’s indicates compactification.

• What is the storage mechanism? What is storing the data? The answer is a quantum bit known as qbit, which is a two-state system, a quantum two-level system with states $|0\rangle$ and $|1\rangle$ which can be controlled (Das Sarma, 2006, p. 32). Specific examples of physical phenomena that can be expressed in the form of qbit include an electron, instantaneous transportation (Bennett, 1995, p. 24), a polarization of a photon (Bennett, 1995, p. 24) left or right circular polarization, all spin-$\frac{1}{2}$ particle (Bennett, 1995, p. 24) such as electrons, an arbitrary superposition of two atomic states and in theory with quarks up, down, charmed. Additional examples and specifics are in the Theory section next.

The reader naturally links the connection to 1’s and 0’s when given two clearly independent options
Introduction

• Primary References

  ▫ The American Institute of Aeronautics and Astronautics (AIAA) References

  ▫ Physics Today References

  ▫ Physics References

  ▫ Textbook References
  ◦ Nouredine Zettili, Quantum mechanics: Concepts and applications. (John Wiley & Sons, Chichester, West Sussex, UK, 2009), 2nd ed.
Theoretical background

- Examples include photon phase, photon intensity, two atomic states, spin-half particles which are fermions entailing quarks and leptons (e.g. electrons and neutrinos), photon polarization, and an existing D-Wave Computer with electrons of Niobium at temperatures approximating absolute zero.

- The potential of quantum computation is in the application of quantum properties of superposition, interference, entanglement, and uncertainty.

- Problems of decoherence and absence of data compression may be addressed by topology of 2-D non-abelian anyons and non-orthogonal states, respectively.

\[ |\psi\rangle = c_0 |+\rangle + c_1 |-> \]
Theoretical Background

- Examples include photon phase, photon intensity, two atomic states, spin-half particles which are fermions entailing quarks and leptons (e.g. electrons and neutrinos), photon polarization, and an existing D-Wave Computer with electrons of Niobium at temperatures approximating absolute zero.

- Examples of potential qbits include photon phase, photon intensity, two atomic states, spin half particles which are fermions: quarks, leptons: electrons, and neutrinos; polarization of photon and an existing D-Wave Computer with electrons of Niobium at temperatures approximating absolute zero. Applications of quantum mechanics to quantum computation focus on superposition, interference parallel computation paths, entanglement, and uncertainty.
Theoretical Background

- Figures T-1, T-2, and T-3 show existing quantum computer with electrons $e^-$ of Niobium Atoms Nb at temperatures approximating absolute zero $T \sim 0.0$ K.

Figure T-1 D-Wave Computer (Hadhazy, AIAA, 2016, July-August, p. 24)
Theoretical Background

• Figures T-2, T-3, and T-4 show existing quantum computer with electrons e\textsuperscript{-} of Niobium Atoms Nb at temperatures approximating absolute zero T \sim 0 \text{ K}.

Figure T-2 D-Wave Cooling Niobium Atoms (Hadhazy, AIAA, 2016, July-August, p. 25)
Theoretical Background

- Figures T-2, T-3, and T-4 show existing quantum computer with electrons \( e^- \) of Niobium Atoms Nb at temperatures approximating absolute zero \( T \sim 0 \) K.

Figure T-3 D-Wave 128 Qbit Chip (Hadhazy, AIAA, 2016, July-August, p. 23)
Theoretical Background

- **The potential of quantum computation is in the application of quantum properties of superposition, interference, entanglement, and uncertainty.**
- Quantum computation is a vast subject covering everything a computer can do now or in the future applying quantum mechanics.
  - An early definition may be limiting.
    - In Das Sarma, Freedman, and Nayak, *Physics Today* article, **quantum computation is defined as the process of initialization, evolution, and measurement of states** (2006, p. 32 from J. Preskill’s “Lecture notes in quantum computation” at Caltech).
    - A better definition may be needed than those published in *Physics Today* articles.
    - Quantum computation is the application of quantum mechanics to computation, specifically advancements using superposition, entanglement, quantum parallelism, and uncertainty.

- Quantum computation is a process consisting of the evolving development of each state incrementally one by one from the
  - initial state $|ψ_0>$ to a
  - final state $|ψ_1>$=$U(t)|ψ_0>$ with
  - unitary transforms varying with time
  - $U(t): |ψ'>=U(t)|ψ>$, or
  - $|ψ' >= U(t)|ψ>$,
  - or $|ψ'> = U(t)|ψ>$ or
  - $|ψ'> = U(t)|ψ$

- Quantum computation may include the storage and retrieval of data in simultaneous states using superposition. So alternatively, a definition in the early stage of development of a new theory and accompanying technology may focus on its potential. The potential of quantum computation is in the application of quantum properties of superposition, interference, entanglement, and uncertainty is untangled thoroughly in the Detailed Exploration section next.

**Quantum computation is defined as**
the process of initialization, evolution, and measurement of states
Theoretical Background

- Problems of decoherence and absence of data compression may be addressed by topology of 2-D non-abelian anyons and non-orthogonal states, respectively.
- In an article in *Physics Today* (2006), Das Sarma, Freedman, and Nayak address quantum computation problems of decoherence and absence of data compression with topology. Aspects of waves canceled in superposed wave functions, known as interference, may allow control of problems associated with decoherence. Topology provides alternative ways of grouping and viewing qbits.
- Many publications seem to begin the discussion of quantum computation by mentioning the quick computation of cryptographic keys based on factoring large numbers. *Physics Today* articles “Quantum information and computation” (Bennett, 1995) and Aerospace America article “New-age computing” (Hadhazy, 2016, July-August) are no exceptions. This is likely because some of the more promising applications of quantum mechanics to computation include instruction execution, storage, cybersecurity encryption, and authentication using encryption like QKD quantum key description where a hacker will observe a changed state and precise clocks at the subatomic quantum level.
- Quantum mechanics applies to computation naturally since computational hardware, the computer, is becoming smaller and smaller approaching the realm of quantum mechanics and computational software is becoming more probabilistic, again approaching the realm of quantum mechanics. Moore’s observation, sometimes called a law, which every couple of years, half the size and double the speed of chips, may break down due to electron leakage. So there may a possible breakdown of Moore’s Law while trying to cross the barrier and bridge from classical computation to quantum computation.
- Quantum mechanics is more general than classical mechanics, so it contains classical mechanics as a limiting case (Zettili, p.189, §3.8.2). Therefore, this paper shows the application of quantum mechanics more than explaining a measurement of an observable classically.
- Application of quantum mechanics in general may apply to any situation where discreet instead of continuous measurements are found in reality. Like the failure of classical physics to apply to the quantum of energy, radiation including light, particles, particle-wave duality, uncertainty principle and blackbody radiation, the photoelectric effect, atomic stability, and atomic spectroscopy (Zettili, 2009, Chap. 1: Origins of Quantum Physics, pp. 2-54; Shankar, 1994, Chap. 3: All Is Not Well with Classical Mechanics, pp. 107-113), quantum computation is such that quantum mechanics applies when classical mechanics cannot.
- Quantum computation consists of applying a sequence of unitary operator transformations to a state vector $\psi$ or $\Psi$ (Bennett, 1995, p. 24). The process consists of unitizing
  - a state $|\psi_0\rangle$ evolving by a unitary transform operator varying with time $U(t)$ to the final state $U(t)|\psi'\rangle$, such that $H|\psi_0\rangle = U(t)|\psi'\rangle$ (Das Sarma, Freedman, and Nayak, 2006, p32).
- The time dependence may be expressed in the form of a Schrödinger’s picture, a Heisenberg picture, and an Interaction picture. Software applications may build on this ability as discussed next.

So, do the quantum computation transformation operators need to be unitary?
Theoretical Background

• Primary References

  ▫ The American Institute of Aeronautics and Astronautics (AIAA) References

  ▫ Physics Today References

  ▫ Textbook References

• Secondary References

  ▫ Example Applications
Detailed Exploration

- There is success of V&V of flight software by Lockheed Martin and Boeing running D-Wave.
- Quantum software to simulate reality at small scales uses the strength of quantum mechanics, as does incorporating quantum uncertainty as a strength instead of eliminating probability.
- Uncertainty is appropriate for Mars robotics with quantum artificial intelligence as is entanglement of 2 atoms and 2 photon modes for communication
  - $|\psi\rangle = \sqrt{\frac{1}{2}} \exp\left(-\frac{iEt}{\hbar}\right) \left[ \cos\left(\frac{E|\lambda|t}{\hbar}\right) (|\varphi_1\rangle + |\varphi_2\rangle) + i \sin\left(\frac{E|\lambda|t}{\hbar}\right) \frac{\lambda}{|\lambda|} (|\varphi_3\rangle + |\varphi_4\rangle) \right]$, expandable to n-particle states.
- Machine learning of robotics will improve by increasing available data with the eMerge app concept to enable public participation and the quantum search in 2-D, defined by
  - $|\Psi\rangle = \sum_a \psi_1 |\!\!\!\!\!\!\!\!\!\!-A,a\rangle + \psi_2 |\!\!\!\!\!\!\!\!\!\!-B,a\rangle$ with $|a\rangle = \frac{1}{2} |0102\rangle + |0112\rangle + |1102\rangle + |1112\rangle$ for V&V backtesting quantum algorithm.

$|+_A,1_A0_B\rangle$
Detailed Exploration

- There is success of verification and validation of flight software by Lockheed Martin and Boeing running D-Wave.
- Lockheed Martin and Boeing have success with Verification and validation (V&V) running D-Wave (Adam Hadhazy, 2016, July-August, p. 22-27).
- Quantum computation should be more suited for artificial intelligence not only because of sophistication in the underlying mechanics but also in the uncertainty inherent in both quantum mechanics and artificial intelligence. In simplest terms, quantum mechanics, which describes units of energy transfer known as quanta, (derived from Suplee, 1999, pp. 84-85), is applied to quantum computation, (1) for faster storage by bit flips using superposition with objects existing in two different states simultaneously, (2) for faster execution than sequential instruction execution by using quantum uncertainty to evaluate large numbers of possibilities simultaneously, and (3) for faster communication with entanglement of particles, providing immediate interaction.

Fast storage, execution, communication
Detailed Exploration

- Uncertainty is appropriate for Mars drones with quantum artificial intelligence as is entanglement of 2 atoms and 2 photon modes for communication, expandable to $n$-particle states.
- Entanglement is the linking of two states, usually, but not necessarily different. Linking here means a strong correlation in measurement. Entanglement applications exist now because of discovery and experimental results, for example spin-up, with another spin-up particle. Therefore, as a thought experiment, if particles are separated with one on Earth and one put on robotics sent to Mars as shown in Figure D-2, then any change in spin to down, for example, results in the other changing to spin-down on Mars. Entanglement particles are separated by space and interact by something other than signals (Derived from Sarfatti, 1977 in Zukav, 2012, pp. 310-311) and are explained by quantum jumps between states being discrete. In general, entanglement could involve subspace overlapping or states overlapping such that independence is gone (Masood, §7.3 p. 403 during Phys5632Qm2 Class Notes 2017-1-26-Thursday).

Entanglement is the linking of two states, usually, but not necessarily different.
Detailed Exploration

- Starting the application of quantum mechanics to quantum computation entanglement with the calculation of a 2-atom and 2-photon mode system with the Hamiltonian seems appropriate as performed by Masood and Miller (2007) by extending the Jaynes-Cummings Model (1963) for entanglement. The entanglement calculations begin with the Hamiltonian. The Hamiltonian,
  - for two atoms A and B,
  - two modes also A and B
  - with atom A interacting with mode A only and
  - atom B interacting with mode B only, is
  \[ H = H_A + H_B \]
  where the only difference is the subscript
  \[ H_A = \hbar \omega_A N_A + \frac{1}{2} E_A \sigma^z_A + \hbar \kappa_A (a_A^\dagger \sigma^-_A + a_A \sigma^+_A) \]

- A key variable in the Hamiltonian is the strength of interaction between atom A and mode A symbolized by \( S_A \). Perhaps entanglement may be defined as interaction strength.
  - The ground state is \(|-\rangle_A\) and the excited state is \(|+\rangle_A\) for atom A.
  - The atomic energy difference for atom A is \( E_A \).
  - The Pauli matrices are \( \sigma_{\pm} \). The identity matrix is \( I \).
  - The number operator for mode A is \( N_A = a_A^\dagger a_A \).
  - The strength of interaction between atom A and mode A is contained in a dimensionless parameter \( \lambda_A \)
    \[ \lambda_A = \frac{\hbar S_A}{E_A} = \frac{q}{6\hbar} \]
    with the definition of Strength \( S_A \) implied by this equation (2007, p. 4 (6))
    \[ S_A = \frac{q}{6\hbar} E_A \]
    at resonance \( q = \lambda_A \) from Masood and Miller (2014, p. 4) with entropy E & S
    - Entropy \( Y_{A/B} = \sin^2 S t \) for atom – atom entropy (2014, p. 7 (III – 18))
    - Entropy \( Y_{a/b} = \cos^2 S t \) for photon mode entropy (2014, p. 7 (III – 18))

- Extending research of Masood & Miller (2014, Fig. 4, p. 7) by solving for the strength of entanglement \( S \)
  - \( S_{A/B} = (\pi n + \sin^{-1}(\sqrt{\text{Entropy}_{A/B}})) t^{-1} \) atom – atom entropy for interger \( n \)
  - \( S_{a/b} = (2\pi n + \cos^{-1}(\sqrt{\text{Entropy}_{a/b}})) t^{-1} \) photon mode entropy for interger \( n \)

- Operating the Hamiltonian on a state results in the eigenvalues \( E \) and the eigenfunctions with the fully superposed system state at time \( t = 0 \) as \( \psi_{a}(t) \)
  \[ |\psi_{a}(t)\rangle = \frac{1}{\sqrt{2}} e^{-iEt/\hbar} \left[ \cos \left( \frac{E_{1}}{\hbar} t \right) |\varphi_1\rangle + |\varphi_2\rangle \right] - i \sin \left( \frac{E_{1}}{\hbar} t \right) \frac{hS}{\hbar} \left( |\varphi_3\rangle + |\varphi_4\rangle \right) \]

- Consistent with Zettili (2009, 2nd, (9.48)*, p. 496), \( \alpha \) is one or more quantum numbers.
Detailed Exploration

- The wave function containing the ground state and the excited state looks similar to a qbit. Applying the quantum mechanics postulate for an observable, there is a linear Hermitian operator, which is the Hamiltonian in this case; whose eigenvectors form a complete basis. Specifically, there are four vector space bases for the two atoms A or B in ground state – or excited state +, a two mode system with two atoms A or B in ground state – or excited state +, and two photon modes 0 or 1 expressed in the form of

\[
\begin{align*}
|\varphi_1\rangle &= |\text{photon mode 0 or 1}; |\text{atom's ground – or excited +}\rangle_{\text{atom A,B}} |\text{mode}; |\text{state}\rangle_{\text{atom}} \\
|\varphi_1\rangle &= |0; |\text{−}\rangle_A |1; |\text{−}\rangle_A |1; |\text{−}\rangle_A |0; |\text{−}\rangle_B \\
|\varphi_3\rangle &= |0; |+\rangle_A |0; |\text{−}\rangle_B |0; |\text{−}\rangle_A |0; |\text{+}\rangle_B \\
|\varphi_4\rangle &= |0; |\text{−}\rangle_A |0; |\text{+}\rangle_B \\
\end{align*}
\]

The wave of ground state and the excited state is applicable to quantum computation as a qbit.
Detailed Exploration

- The probability of entanglement if a photon in mode A or B is absorbed or emitted by an atom in state $| - \rangle$ or $| + \rangle$, respectively (Samina S. Masood and Allen Miller, 2007, p. 3) is the square of the modulus of the coefficients of the wave function $|\psi_{\alpha}(t)\rangle$

- $$P_1 = \left| \sqrt{\frac{1}{2}} e^{-iE_1 t/\hbar} \ast \cos \left( \frac{E_1 h S_1}{E_1} t \right) \right|^2$$
  $$= \frac{1}{2} \cos^2 (S_1 t)$$
  for $|\varphi_1\rangle = |0; -\rangle_A |1; -\rangle_A$
  for real $E$ and $S$

- $$P_2 = P_1$ with the subscript 2 for $E$ and $\kappa$$
  $$= \frac{1}{2} \cos^2 (S_2 t)$$
  for $|\varphi_2\rangle = |1; -\rangle_A |0; -\rangle_B$
  for $|\varphi_2\rangle = |1; -\rangle_A |0; -\rangle_B$

- $$P_3 = \left| \sqrt{\frac{1}{2}} e^{-iE_3 t/\hbar} \ast i \sin \left( \frac{E_3 h S_3}{E_3} t \right) \right|^2$$
  $$= \frac{1}{2} \sin^2 (S_3 t)$$
  for $|\varphi_3\rangle = |0; +\rangle_A |0; -\rangle_B$
  for real $E$ and $S$

- $$P_4 = P_3$ with the subscript 4 for $E$ and $\kappa$$
  $$= \frac{1}{2} \sin^2 (S_4 t)$$
  for $|\varphi_4\rangle = |0; -\rangle_A |0; +\rangle_B$
  for $|\varphi_4\rangle = |0; -\rangle_A |0; +\rangle_B$

The probability of entanglement if a photon in mode A or B is absorbed or emitted
Detailed Exploration

• As summarized by (Samina S. Masood and Allen Miller, 2007, p. 10), if atom A is in a ground state, then we know, with certainty, that atom B is excited, and vice versa. In other words, it is impossible for both atoms to be in their ground states, at time $t = t_0$. There is a **periodic** increase and decrease of the strength $S$ of the entanglement, as expressed by the equation for $|\psi_\alpha(t)\rangle$ above. The period $P$ for a full cycle of the strength $S$ of the entanglement $S(t)$ is

\[
P = \frac{2\pi \hbar}{E_{atom}} \frac{S_{atom}}{E_{atom}}
\]

for real $E$ and $S$

• **Entanglement is not a function of distance.** So, continuing the thought experiment stated with Figure D-2, if one entangled atom is on Earth and the other entangled atom is with the Mars robotics, a change in one atom may be observed as the opposite change in the other in the case of a 2-atom and 2-photon mode system. Since the calculation and equation for $|\psi_\alpha(t)\rangle$ above is expandable to $n$-particle states, communications from Earth to Mars is expandable to $n$-qbits, with one system instead of $n$-separate systems.

• The timing of communication can be aligned with entanglement. The periodicity of the strength of entanglement can be used to determine the best timing for communication. Simply knowing which two particles are entangled, without knowing why, may seem sufficient to apply quantum mechanics to quantum computation, but since strength of entanglement varies with time, knowing why two particles are entangled becomes key to knowing the periodicity of the strength of entanglement. Two particles may be observed by experiment to be entangled. Through calculations in this paper, knowing why two particles are entangled becomes clear. Specifically, both probability and strength of entanglement are **periodically** oscillating functions of time, so there are best times to measure entanglement. This timing of measurement becomes useful in practical application for public participation in space exploration.

Entanglement is not a function of distance.

So if one atom is on Earth and the other atom is with the Mars robotics, a change in one atom may be observed in the other.
Detailed Exploration

- Machine learning of robotics will improve by verification and validation backtesting of quantum algorithms with (1) increasing available data through the eMerge app concept to enable public participation and (2) using the quantum search in 2-D.
- For example, the eMerge app concept could increase data availability to improve machine learning of robotics in space. The eMerge app consists of five levels of engagement (Alex Monchak, Ki-Young Jeong, and James C. Helm, 2013, p. 1-19), each with a separate app, with increasing engagement capabilities for additional cost. The five separate apps are Observation, Interaction, Participation, Empowerment, and Competition as shown in Figure D-3 with a picture logo and a word logo.

With the establishment of the professional aerial Drone Racing League (DRL), Mars drone races via eMerge will likely require machine learning unless entanglement speeds communication back to the drone operators on Earth.
Detailed Exploration

- For a specific example of quantum parallelism since energy states and nuclear spin states commute, the quantum search in two-dimensions (Chamoli & Masood, 2010, p. 4) is defined by the superposition of all possible states in the database
  \[ |\Psi\rangle = \sum_{a=0}^{N} \psi_1(a)|-A, a\rangle + \psi_2(a)|-B, a\rangle \quad \text{for } N \text{ number of entries in the data base} \]

- For \( |a\rangle \) representing the superposed nuclear spin states of atoms A and B
  \[ |a\rangle = \frac{1}{2} (|0_A 0_B\rangle + |0_A 1_B\rangle + |1_A 0_B\rangle + |1_A 1_B\rangle) \]

- For the ground state as – and the excited state as +
  \[ |\Psi\rangle_a = \frac{1}{2} [\psi_A(a)(|-A, 0_A 0_B\rangle + |-A, 0_A 1_B\rangle + |-A, 1_A 0_B\rangle + |-A, 1_A 1_B\rangle) + \psi_B(a)(|-B, 0_A 0_B\rangle + |-B, 0_A 1_B\rangle + |-B, 1_A 0_B\rangle + |-B, 1_A 1_B\rangle)] \]

- To demonstrate different sizes of information in database fields for a 2-D quantum search, there are two values for the first search term and four values for the second search term. A successful search occurs when all database entries are identified with atom A in an excited state + and nuclear spin states of atom A down, symbolized by 1, and atom B up, symbolized by 0, so the state of each found entry is of the form
  \[ |+_{A, 1_A 0_B}\rangle \]

- The initial superposition state representing the items in the database is
  \[ |\Psi\rangle = \frac{1}{\sqrt{N}} \sum_{a=0}^{N} |a\rangle \quad \text{for } N \text{ number of entries in the data base} \]

- A photon in mode A excites atom A when the first search term is found. Energy states and nuclear spin states commute. Therefore, simultaneously a quantum phase flip operator inverts the spin when the second search term is found. Then another quantum operator repeatedly conducts an inversion about the mean of the amplitude of all base states to provide amplitude amplification, increasing the probability for more accurate measurement of a successful search. Prime indicates the updated state
  \[ |\Psi\rangle_a' = \frac{1}{2} [\psi_A(a)(|+_{0_A 0_B}\rangle + |+_{0_A 1_B}\rangle + |+_{1_A 0_B}\rangle + |+_{1_A 1_B}\rangle) + \psi_B(a)(|-_{0_A 0_B}\rangle + |-_{0_A 1_B}\rangle + |-_{1_A 0_B}\rangle + |-_{1_A 1_B}\rangle)] \]

- Bold indicates successful search state

- The measurement of the atom A’s excited state with a measurement of nuclear spin states reveals the database entries containing the two search terms. The quantum parallelism in quantum search in two-dimensions may be extended to more dimensions, which are the search terms, and should improve machine learning of robotics when used for the verification and validation (V&V) backtesting of quantum algorithm. Quantum mechanics improves machine learning by the potential to improve storage speed and instruction execution speed, especially needed for robotics on Mars.

For a specific example of quantum parallelism since energy states and nuclear spin states commute is the quantum search in two-dimensions (Chamoli & Masood, 2010, p. 4)
Detailed Exploration

Primary References

The American Institute of Aeronautics and Astronautics (AIAA) References


Physics Today References


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Textbook References

- Nouredine Zettili, Quantum mechanics: Concepts and applications. (John Wiley & Sons, Chichester, West Sussex, UK, 2009), 2nd ed.

Secondary References

Example Applications


Additional Secondary References:

- Alex Monchak and Samina S. Masood, “Binary Star Model in the physics seminars with Samina S. Masood, PhD (Faculty sponsor)”, in Physics Seminar at the 20th Annual Student Conference for Research and Creative Arts (RAC) (University of Houston - Clear Lake, Houston, TX, 2014, April 15).
- Alex Monchak and David Garrison, “Narrated visuals, pictures, discipline reports, demonstrations and working meetings in physics seminars with David Garrison, PhD (Faculty sponsor)”, in 15th Annual Student Conference for Research and Creative Arts (RAC) (University of Houston - Clear Lake, Houston, TX, 2009, April 22-23).
**Brief summary**

- Like the acceptance of new and existing physics equations and the eMerge app, Ease of use and Usefulness lead to quantum computation acceptance and associated acceleration.

  ▫ A summary of this paper may be expressed as justification for realizing that quantum computation is still in the early stages of technology development and acceptance by potential users.

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**Acceleration of quantum computation acceptance**

\[ a = \frac{d^2A}{dt^2} \]
Brief Summary

- Like the acceptance of new and existing physics equations and the eMerge app, Ease of use and Usefulness lead to quantum computation acceptance and associated acceleration.

- A summary of this paper may be expressed as justification for realizing that quantum computation is still in the early stages of technology development and acceptance by potential users. Since quantum mechanics successfully matches observations at the small, fast, and encrypted scale (and matches the large, less fast, and less encrypted scale in the limiting case) and computers are becoming smaller, faster, and more encrypted (and mathematics is a tool used to increase precision in quantum mechanics); quantum mechanics should apply to computation, resulting in quantum computation.

- Similar to how Ease of use and Usefulness of any physics equation and apps like eMerge determine their acceptance, the same is true for quantum computation. Specifically, Ease of use and Usefulness of quantum computation lead to its acceptance.

Quantum computation is still in the early stages of technology development
As is discussed in this paper, developments in quantum computation are proceeding. New technologies often transition from prior technologies building on prior advances. New technologies may also disrupt prior technologies. Prior technologies are often in the later stages of acceptance when the transition or disruption occurs. The developments in quantum computation may lead to transition, disruption, or both as shown in the Figure.
Acceptance of quantum computation depends on 

Ease of use $E$  \[ A = E + U \]

Usefulness $U$  \[ = w_1E + w_2U \]

Conceptually

Specifically

More Specifically

Acceptance of Quantum Computation

Attitude leads to intention to use, which in turn leads to acceptance of quantum computation.
Ease of use ranges from 0 to 1. Zero is no Ease of use, while 1 is completely easy to use.
Usefulness also ranges from 0 to 1.
Zero is no Usefulness, while 1 is completely Usefulness.
Each is equality weighted and contribute 50% to Acceptance, which also ranges from 0 to 1.
Summary

- Acceptance $A$ of quantum computation also changes with time $t$
  
  $A = f(E, U, t)$

- For Acceptance $A$ represented by a specific type of S-curve in time $t$ known as a sigmoid curve
  
  $A = (1 + e^{-t})^{-1}$

- The velocity $v$ of acceptance is the first derivative with respect to time $t$
  
  $v = \frac{d}{dt} A$
  
  $= \frac{d}{dt} (1 + e^{-t})^{-1}$
  
  $= e^t (1 + e^t)^{-2}$

- The acceleration $a$ of acceptance is the second derivative with respect to time $t$
  
  $a = \frac{d^2}{dt^2} A$
  
  $= \frac{d^2}{dt^2} (1 + e^{-t})^{-1}$
  
  $= \frac{d}{dt} e^t (1 + e^t)^{-2}$
  
  $= e^t (1 - e^t)(1 + e^t)^{-3}$

Acceptance of quantum computation changes with time
Acceptance of quantum computation is still early. This paper supports identification of the current status of quantum computation at the lower knee of the acceptance S-Curve.
Acceptance of quantum computation is approaching maximum acceleration. Early acceptance difficulties are discussed in the literature without focusing on the mathematical observation and physical limitations of increasing acceleration to maximum acceleration, which may be causing the underlying explanation of these difficulties.

It is like riding a bike too fast when you are just learning how to ride.
Diffusion of quantum computation

As acceptance of quantum computation approaches maximum acceleration, often innovations fail to cross over the chasm to achieve larger acceptance with early adopters.
Acceptance of quantum computation

Lee Morin, Sandra Magnus, Stanley Love, Donald Pettit, and Mary Lynne Dittmar (*Lunar Settlements*, 2010), may not have anticipated the use of quantum computation for initial robotic lunar resource development. But the importance of a mobile application concept such as eMerge should help.

Hopefully quantum computation crosses the diffusion chasm.
Summary

Future Research: Experiment

- The paper builds from hardware to software to acceptance of quantum computation. This leads to consideration of building and testing software applications. For further study, consider an educational research experiment focusing on an application software development with remote access to the D-Wave at Boeing or Lockheed.

- An extension of this research focusing on entanglement is also hopeful. More rigorous research is in order. Dr. Masood’s entanglement research, which extends the Jaynes-Cummings Model in an optical cavity to include entanglement, may be extended further. This extension is a more demanding analysis. The analysis should consist of four sections.

  - Section 1: Definition of Entanglement such as extending this paper’s entanglement definition as interaction strength to flow form quantum mechanics to entanglement to mathematical calculations. Then
  - Section 2: Summary of Jaynes-Cummings Model in an optical cavity.
  - Section 3: Calculations:
    - Calculate cavity loss by a straightforward calculation or
    - calculate an $n$-particle extension for an $n$-body system or
    - combine both to calculate the cavity loss in a laser cavity for an $n$-particle extension for an $n$-body system by incrementing the number of qbits in a single state or
    - calculate the change in energy with the change in time for possibly 2-modes or
    - calculate the energy losses corrected with electromagnetic (EM) entanglement since some energy is not losted and
    - expanding research with access to the D-wave quantum computer is a bonus by extending the current confirmation of entanglement (Lanting et. Al, 2014) to $n$-particle entanglement for an arbitrary $n$. Finally,
  - Section 4: Conclusion Drawn.

Access to the D–Wave: Extending the current confirmation of entanglement
Summary References


Quantum computation is envisioned as the evolution of a system from an initial state to a final state by a unitary transform operator varying with time and the states are the form of a qbit. Anything able to be expressed as at least two states in the form of a qbit may be used as the hardware for quantum computation. Examples include photon phase, photon intensity, two atomic states, spin-half particles which are fermions entailing quarks and leptons (e.g. electrons and neutrinos), photon polarization, and an existing D-Wave Computer with electrons of Niobium at temperatures approximating absolute zero. The potential of quantum computation is in the application of quantum properties of superposition, interference, entanglement, and uncertainty. Problems of decoherence and absence of data compression may be addressed by topology of 2-D non-abelian anyons and non-orthogonal states, respectively. There is success of verification and validation of flight software by Lockheed Martin and Boeing running D-Wave. Uncertainty is appropriate for Mars drones with quantum artificial intelligence as is entanglement of 2 atoms and 2 photon modes for communication, expandable to \( n \)-particle states. Machine learning of robotics will improve by verification and validation backtesting of quantum algorithms with (1) increasing available data through the eMerge app concept to enable public participation and (2) using the quantum search in 2-D. Like the acceptance of new and existing physics equations and the eMerge app, Ease of use and Usefulness lead to quantum computation acceptance and associated acceleration.
Question, Answers, & Suggestions?

\[ \psi = \sum_{x=00...0}^{11...1} c_x |x\rangle \]

\[ |+_A, 1_A 0_B\rangle \]