Direct Transfers From Selenocentric Equatorial Orbit To The Lunar Surface

Consider a circular selenocentric equatorial orbit (SEO) of radius r_1. An idealized direct lunar landing strategy is to be assessed by this paper as r_1 is systematically varied. This strategy uses two impulses as follows.

1) The first "deorbit" impulse Δv_1 rotates the SEO plane through the angle Δi and lowers pericynthion such that the desired landing site is intercepted after coasting through a selenocentric transfer angle of 90°. Assuming the landing site is at selenocentric latitude ϕ, $\Delta i = | \phi |$.

2) The second "landing" impulse Δv_2 stops inertial selenocentric motion at landing site intercept. Note maximum inertial motion of the Moon's surface at its equator is 4.6 m/s. Such motion is assumed insignificant at the level of this assessment's fidelity.

The selenocentric post-deorbit coasted trajectory to landing site intercept is assumed to describe an unperturbed ellipse segment starting from apocynthion at r_1. In this assessment, the SEO may be prograde or retrograde without affecting results. Furthermore, the landing strategy's two impulses may be reversed to create an equivalent launch strategy without affecting assessment results. A roundtrip from SEO at a particular r_1 to the specified landing site at ϕ is therefore just twice the assessed $\Delta v_{TOT} = \Delta v_1 + \Delta v_2$. Physical constants of relevance to this assessment are the Moon's reduced mass $\mu = 4902.80007$ km^3/s^2 and the Moon's surface radius $R = 1737.4$ km.[1]

Assessment computations fundamentally depend on the polar expression for conic trajectories in Equation 1. This relation determines selenocentric distance r as a function of true anomaly ν in the post-deorbit ellipse specified by its fixed eccentricity e and its semi-latus rectum p.

$$r = \frac{p}{1 + e \cos \nu} \tag{1}$$

As the coast from deorbit to landing proceeds, ν increases from 180° to 270°. Consequently, the landing case for Equation 1 produces $r = p = R$. The deorbit case for Equation 1, when solved for e, leads to Equation 2.

$$e = 1 - \frac{R}{r_1} \tag{2}$$

Since r_1 is at apocynthion, the coast trajectory's semi-major axis a is determined by Equation 3 in accord with Kepler's first law and ellipse geometry.

$$a = \frac{r_1}{1 + e} \tag{3}$$

[1] Values for these constants are obtained from the *Horizons* ephemeris server at https://ssd.jpl.nasa.gov/?horizons (accessed on 23 March 2017).

The energy integral for conic motion leads to expressions for selenocentric speed immediately before and after the deorbit impulse.

$$v_{1-} = \sqrt{\frac{\mu}{r_1}} \tag{4}$$

$$v_{1+} = \sqrt{\mu\left(\frac{2}{r_1} - \frac{1}{a}\right)} \tag{5}$$

Now consider a velocity vector triangle with Δv_1 as its unknown side. Known sides v_{1-} and v_{1+} for this triangle are legs of the specified deorbit turning angle Δi. When the law of cosines is applied to the triangle, Equation 6 results.

$$\Delta v_1 = \sqrt{v_{1-}^{\;2} + v_{1+}^{\;2} - 2\, v_{1-}\, v_{1+} \cos \Delta i} \tag{6}$$

The conic motion energy integral also determines the landing impulse.

$$\Delta v_2 = \sqrt{\mu\left(\frac{2}{R} - \frac{1}{a}\right)} \tag{7}$$

Results from numerically evaluated landing strategy assessments with Δi = 90° appear in Table 1. Simulations of the post-deorbit coast for each assessment contribute Table 1 data for the coast time from deorbit to landing Δt, together with the inertial flight path angle at landing γ_2. Both Δv_{TOT} and Δt values from Table 1 are plotted as functions of r_1 in Figure 1. Coasted inertial selenocentric motion arising from the r_1 = 10,000 km simulation is plotted in Figure 2.

Table 1. One way direct transfers from SEOs of increasing radii are assessed with Equations 2-7 for polar lunar landings (Δi = 90°). The second r_1 = 70,000 km assessment (in *italics*) has a Δv_1 whose Equation 4 v_{1-} contribution is overridden with zero to approximate best-case deorbit initial conditions from a periodic orbit about the Earth-Moon cislunar (EML1) or trans-lunar (EML2) colinear libration points.

r_1 (km)	Δv_1 (km/s)	Δv_2 (km/s)	Δv_{TOT} (km/s)	Δt (hours)	γ_2 (deg)
10,000	0.758592	2.179095	2.937687	4.836	-39.566
20,000	0.516174	2.274830	2.791004	13.119	-42.400
30,000	0.415802	2.307910	2.723712	23.729	-43.292
40,000	0.357622	2.324655	2.682277	36.235	-43.728
50,000	0.318533	2.334766	2.653299	50.381	-43.987
60,000	0.289965	2.341533	2.631497	65.997	-44.158
70,000	0.267915	2.346379	2.614294	82.956	-44.280
70,000	*0.041694*	*2.346379*	*2.388073*	*82.956*	*-44.280*

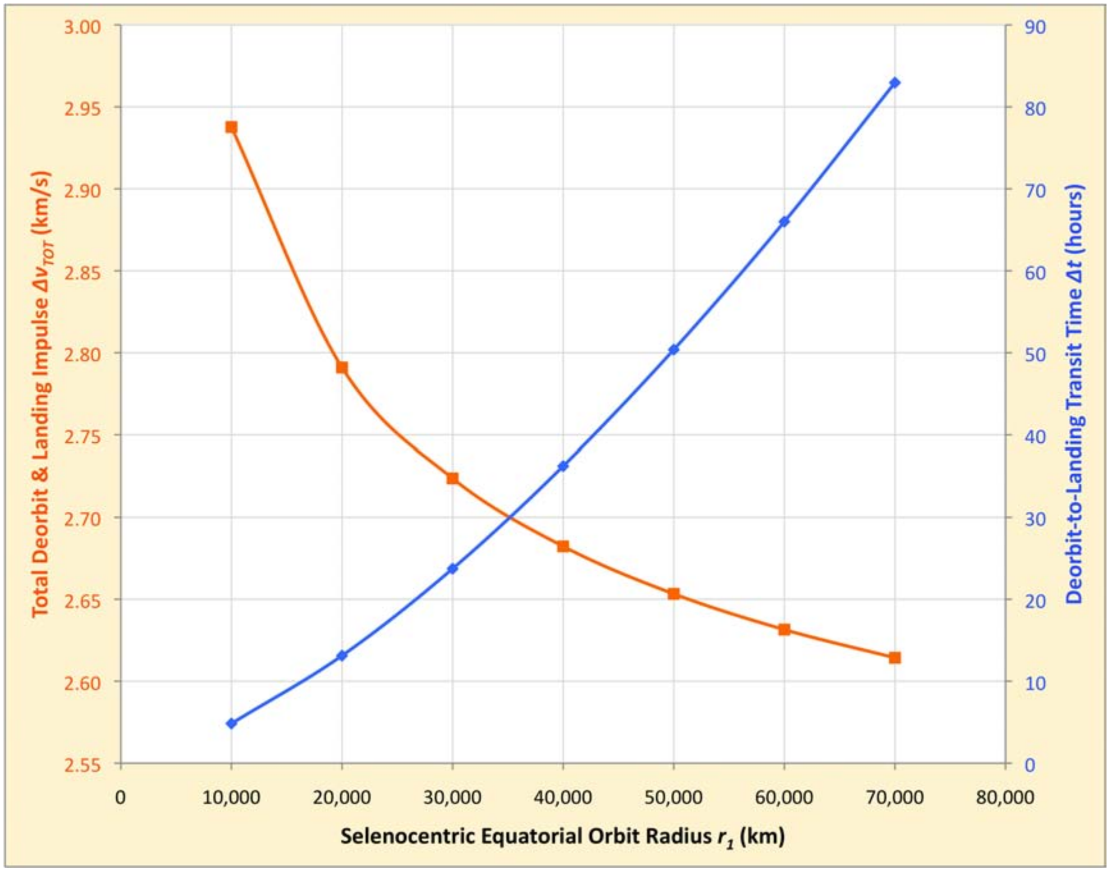

Figure 1. Table 1 data for Δv_{TOT} (orange) and Δt (blue) are plotted as functions of r_1.

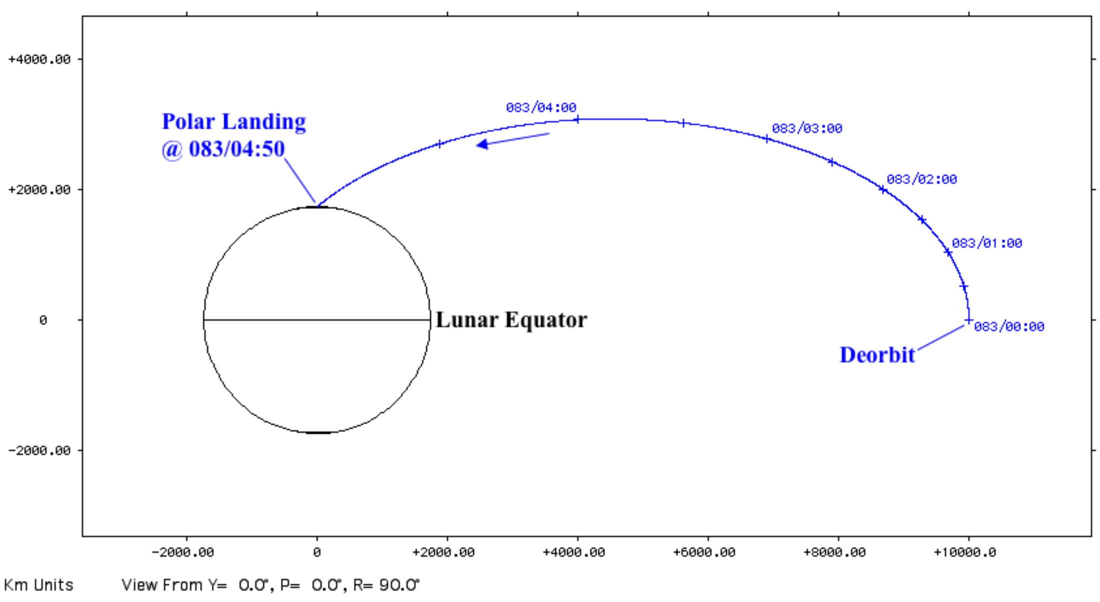

Figure 2. Coasted post-deorbit inertial selenocentric motion from r_1 = 10,000 km to a polar landing is plotted for a lunar equatorial perspective. With Δi = 90°, pre-deorbit SEO motion is perpendicular to the plot plane. Time ticks along the coasted trajectory are at 30-minute intervals and are annotated on the even hour in *day-of-year/hh:mm* format.

The foregoing assessments are conducted at Δi = 90° because the Moon's polar regions are widely considered to be prime locations for lunar in-situ resource utilization (ISRU). Many cislunar architecture proposals advocating lunar ISRU logistics nodes and gateways are orbit-agnostic. Still other proposals limit SEO considerations to radii near 70,000 km in the Moon's weak stability boundary, likely because NASA has targeted such an orbit as the destination for its Asteroid Retrieval Robotic Mission.

Under simplifying assumptions of this paper, polar landing is certainly a maximum Δv_{TOT} case for any SEO, regardless of r_1. Many periodic orbits associated with EML1/L2 are similarly challenged when a landing at $\phi = \pm 90°$ is contemplated. Table 1 data quantify, at least to first order, trends in propulsive requirements and transit times associated with polar lunar surface logistics to and from SEOs. Because EML1/L2 are near a selenocentric distance of 70,000 km, the two Table 1 assessments at r_1 = 70,000 km serve to place at least some polar landing cases from EML1/L2 periodic orbits in the interval 2.38 km/s < Δv_{TOT} < 2.62 km/s. Polar landing from an r_1 = 10,000 km SEO therefore requires 12% to 23% more Δv_{TOT} than does a representative case from EML1/L2 periodic orbits. In contrast, landing from an r_1 = 10,000 km SEO requires only 6% of the time required to land from periodic orbits near EML1/L2. Considerations, such as whether or not humans are landed on the Moon, will heavily influence the premiums placed on propulsive requirements and transit times associated with candidate cislunar logistics node orbits.