Earth Orbits With Repeating Ground Tracks

1. Introduction

This paper documents how to compute low Earth orbit (LEO) apsis heights H_A x H_P at which coasted motion will produce a repeating ground track on Earth's surface at true inclination *i* over *k* complete orbits, where *k* is an integer. Because gravity perturbations from excess mass about Earth's equator are considered by this documentation, a "complete orbit" should be regarded as spanning the time interval P_Ω between successive ascending node passages on Earth's equator[1].

Repeating ground tracks are of great utility in spaceflight. For mission objectives relating to Earth observation, ground track repetition enables temporal comparisons between observable changes at particular locations. Arguably one of the most demanding ground track repetition requirements arose from the second Space Radar Laboratory (SRL-2) payload flown aboard Space Shuttle *Endeavour* during the STS-68 mission. In this flight, radar observations made on one day would be combined with those made of the same terrain on another day from a slightly different perspective to produce precise 3-dimensional maps through interferometric analysis.[2] The necessary coherence between these SRL-2 observations was achieved by duplicating trajectories relative Earth's surface to within a few hundred m.

A repeating ground track for a LEO destination such as the International Space Station (ISS) is equivalent to a repeating rendezvous phase angle θ for logistics vehicles launching to visit it. A geocentric angle, θ is defined with its legs directed at ISS and at the logistics vehicle's position projected into the ISS orbit plane. Thus, $\theta = 0$ at rendezvous. To achieve ISS rendezvous in less than 12 hours, as is now common practice for crewed *Soyuz* launches, requires θ in a very narrow range at liftoff. Consequently, if ISS has a repeating ground track with acceptable θ on one day, it will offer an equally acceptable launch opportunity *k* orbits later.

2. Formulation

The following physical constants are relevant to analysis of LEOs with repeating ground tracks. Similar analyses have undoubtedly been made in other planetocentric contexts using appropriate values for these constants.

μ ≡ Earth's gravitational parameter = 398,600.4415 km^3/s^2 [1, p. 7]
R ≡ Earth's mean equatorial radius = 6378.1363 km [1, p. 18]
J_2 ≡ Earth's unnormalized second degree zonal gravity coefficient = 0.0010826269 [1, p. 19][3]
ω ≡ Earth's inertial rotation rate = 7.292115*10^{-5} rad/s [1, p. 19][4]

[1] Although this paper only considers a complete orbit to start and end at successive *ascending* nodes, an equally valid computation could be performed considering a complete orbit to start and end at successive *descending* nodes.
[2] Reference https://www.nasa.gov/mission_pages/shuttle/shuttlemissions/archives/sts-68.html (accessed 23 April 2017).
[3] The degree to which excess mass is distributed about Earth's equator in a manner departing from spherical symmetry is quantified by J_2.
[4] An Earth sidereal day is $2\pi/\omega$ = 86,164.101 s = 23 hrs 56 min 4.101 s.

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Ignoring LEO perturbations from J_2 for the moment, Kepler's third law permits the conic LEO period P_K to be computed from the orbit's semi-major axis $a = R + 0.5\,(H_A + H_P)$ in terms of the associated mean motion n [1, Eqn. 10-20, p. 764].

$$P_K = \frac{2\,\pi}{n} = 2\,\pi\sqrt{\frac{a^3}{\mu}} \tag{1}$$

Assuming a nearly circular LEO, such that eccentricity $e = (a - R - H_P)/a$ is on the order of J_2 or less, a concise approximation for the nodal period spanning a complete orbit can be derived to include the influence of J_2 [1, Eqn. 10-22, p. 765].

$$P_\Omega = P_K\left[1 - 1.5\,J_2\left(\frac{R}{a}\right)^2\left(3 - 4\,\sin^2 i\right)\right] \tag{2}$$

The rate $\dot{\lambda}$ at which Earth longitude shifts under the LEO has two inertial components. By convention, these components are signed such that an eastward shift is positive, and a westward shift is negative. The first component is Earth's eastward rotation $\omega > 0$, and the second component is the LEO's nodal regression rate $\dot{\Omega}$ arising from J_2 perturbations. Per Equation 3 [1, Eqn. 8-37, p. 581], the sign of $\dot{\Omega}$ depends on the sign of -cos i. Thus, prograde LEOs have a westward $\dot{\Omega} < 0$. Note Equation 3 is also dependent on the LEO's semi-latus rectum $p = a\left(1 - e^2\right)$ [1, Eqn. 1-11, p. 14].

$$\dot{\Omega} = -1.5\,\frac{n\,R^2\,J_2}{p^2}\cos i \tag{3}$$

Figure 1 illustrates how $\dot{\Omega}$ varies with i for a LEO with $e = 0$ and height $H = H_A = H_P = +400$ km.

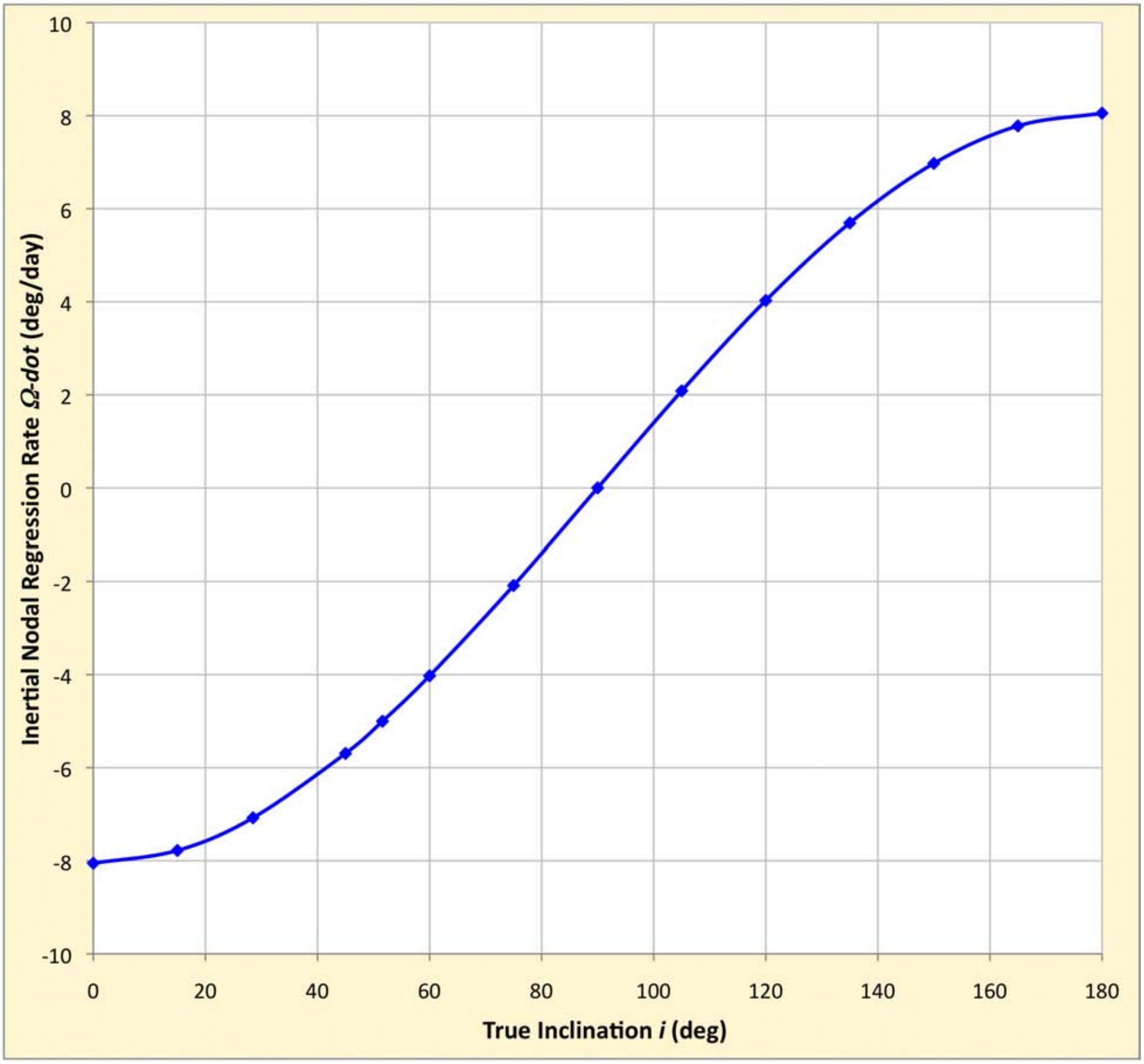

Figure 1. Variations in $\dot{\Omega}$ as a function of *i* are plotted for *e* = 0 and *H* = +400 km.

The two inertial rates comprising $\dot{\lambda}$ must be properly differenced to obtain LEO east/west drift with respect to Earth's surface.

$$\dot{\lambda} = \dot{\Omega} - \omega \tag{4}$$

Now consider the Earth longitude shift over k complete orbits expressed as the number of cycles σ through 2π radians. The criterion for a repeating ground track requires σ from Equation 5 be an integer.

$$\sigma = \frac{k\, P_{\Omega}\, \dot{\lambda}}{2\,\pi} \tag{5}$$

In practice, k is selected to drive σ to a nearly integral value for a specified i and preliminary a reflecting operationally desirable H_A x H_P values. Iterations on a then achieve an integer σ condition to acceptable precision.

3. Examples

Numeric examples in Subsections 3.1 and 3.2 assume circular LEOs with $e = 0$, orbit height H, and orbit radius $r = H + R = a = p$. This assumption is consistent with the low e pedigree for Equation 2. Numeric simulations verifying the precision of $H_A = H_P$ and ground track repetition are performed using the WeavEncke orbit predictor [2][5]. Unless noted otherwise, WeavEncke models Earth gravity with J_2 as the only perturbation to circular LEO motion in these simulations.

Simulations are initialized with geocentric inertial position $\boldsymbol{r}$ and velocity $\boldsymbol{v}$ vectors using a right-handed Cartesian coordinate system whose first vector component is aligned with the direction to 0 Latitude; 0 Longitude. The second vector component in this system is aligned with the direction to 0 Latitude; 90° E Longitude, and the third vector component is aligned with the direction to 90° N Latitude. In practice, $\boldsymbol{r} = [r, 0, 0]^T$. A preliminary value for initial speed $v = \sqrt{\mu / r}$ is provided by the *vis viva* or energy integral [1, Eqn. 2-12, p. 110] of conic motion. The preliminary $\boldsymbol{v} = [0, v \sin i, v \cos i]^T$ ensures simulation initialization at ascending node longitude $\lambda_{AN} = 0$ and the specified i. This preliminary $\boldsymbol{v}$ does not account for J_2's excess equatorial mass because v assumes unperturbed conic motion. Therefore, preliminary v is slightly too small and will require iteration to achieve $H_A = H_P$. Simulation values provided for H_A and H_P in Subsections 3.1 and 3.2 are J_2-aware [3].

3.1 SRL-2 Interferometry During STS-68

To provide maximum permissible geographic coverage for this Earth mapping mission, $i = 57°$ was required. In addition, H was maintained near +222 km to achieve adequately strong radar echoes.[6] Commencing with these mission values, $k = 15$ produces $\sigma = -0.941$ and $k = 16$ produces $\sigma = -1.004$, indicating H was selected for this i to produce a repeating ground track every day. With $k = 16$ adopted for this example, iterating H to +204.6 km produces $\sigma = -0.999994$.

The preliminary $v = 7.781542$ km/s for $H = +204.6$ km produces H_A x $H_P = +204.6$ x $+193.9$ km at simulation initialization. After a 16-orbit coast, the preliminary case achieves $\lambda_{AN} = 0.563°$ E, confirming v is too small because Earth has insufficient time to rotate eastward during those 16 complete orbits. Iterating to $v = 7.785702$ km/s produces initial H_A x $H_P = +208.0$ x $+204.6$ km and $\lambda_{AN} = 0.000°$ E after 16 complete orbits. Consequently, $H = 0.5\ (208.0 + 204.6) = +206.3$ km appears to be the 1-day ground track repetition circular LEO at $i = 57°$. This is confirmed

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[5] Note corresponding physical constants in the WeavEncke orbit predictor differ slightly from those introduced in Section 2 of this paper. From numeric results presented herein, these differences do not appear significant.
[6] Reference https://www.nasa.gov/mission_pages/shuttle/shuttlemissions/archives/sts-68.html (accessed 23 April 2017).

with simulations iterating to v = 7.783689 km/s, producing initial H_A x H_P = +206.3 x +206.3 km and λ_{AN} = 0.000° E after 16 complete orbits.

At H near +206.3 km, STS-68 was flying one of the lowest LEOs of the Space Shuttle Program[7]. This flight regime encounters high atmospheric drag accelerations acting to quickly disrupt a repeating ground track. As an example, the H = +206.3 km case with initial v = 7.783689 km/s is coasted 16 complete orbits with ballistic aerodynamic drag modeled using STS-68 flight data from the author's personal archives as follows.

Endeavour mass = 104,000 kg
Endeavour area for drag coefficient of 2 = 120 m^2
Time-averaged solar flux at 10.7 cm wavelength = 79 Jansky*10^4
Time-averaged geomagnetic index = 2.69

Following the ballistic drag coast, λ_{AN} = 0.058° E and H_A x H_P = +204.9 x 204.6 km. The shift in λ_{AN} from the desired null value is equivalent to 6.4 km east on Earth's surface, rendering SRL-2 interferometry inoperable. During STS-68, altitude decay had to be addressed with pairs of prograde impulses prior to conducting interferometry on a particular crew day.

3.2 ISS

Regular prograde "reboosts" are performed by ISS to maintain mean H near +400 km and to manage rendezvous phase angle for visiting logistics vehicle launch opportunities. At i = 51.6° and H = +400 km, there are no repeating ground track conditions near one day (k = 15 produces σ = -0.979, k = 16 produces σ = -1.044) or near two days (k = 30 produces σ = -1.959, k = 31 produces σ = -2.024). The shortest repetition in the ISS context is near 3 days (k = 45 produces σ = -2.938, k = 46 produces σ = -3.004). With k = 46 adopted for this example, iterating H to +394.5 km produces σ = -3.000080.

The preliminary v = 7.671672 km/s for H = +394.5 km produces H_A x H_P = +394.5 x +383.0 km at simulation initialization. After a 46-orbit coast, the preliminary case achieves λ_{AN} = 1.343° E, confirming v is too small because Earth has insufficient time to rotate eastward during those 46 complete orbits. Iterating to v = 7.674955 km/s produces initial H_A x H_P = +394.6 x +394.5 km and λ_{AN} = 0.000° E after 46 complete orbits. Figure 2 confirms ISS was operating 6 to 11 km above the k = 46 ground track repetition H during much of years 2015 and 2016. This additional mean orbit height would cause λ_{AN} to drift slightly westward after each 3-day cycle.

[7] As-flown STS-68 data in the author's personal archives confirm H_A x H_P = +204.7 x +202.9 km 10.95 days after launch (following SRL-2 operations). Deorbit was performed 11.20 days after launch.

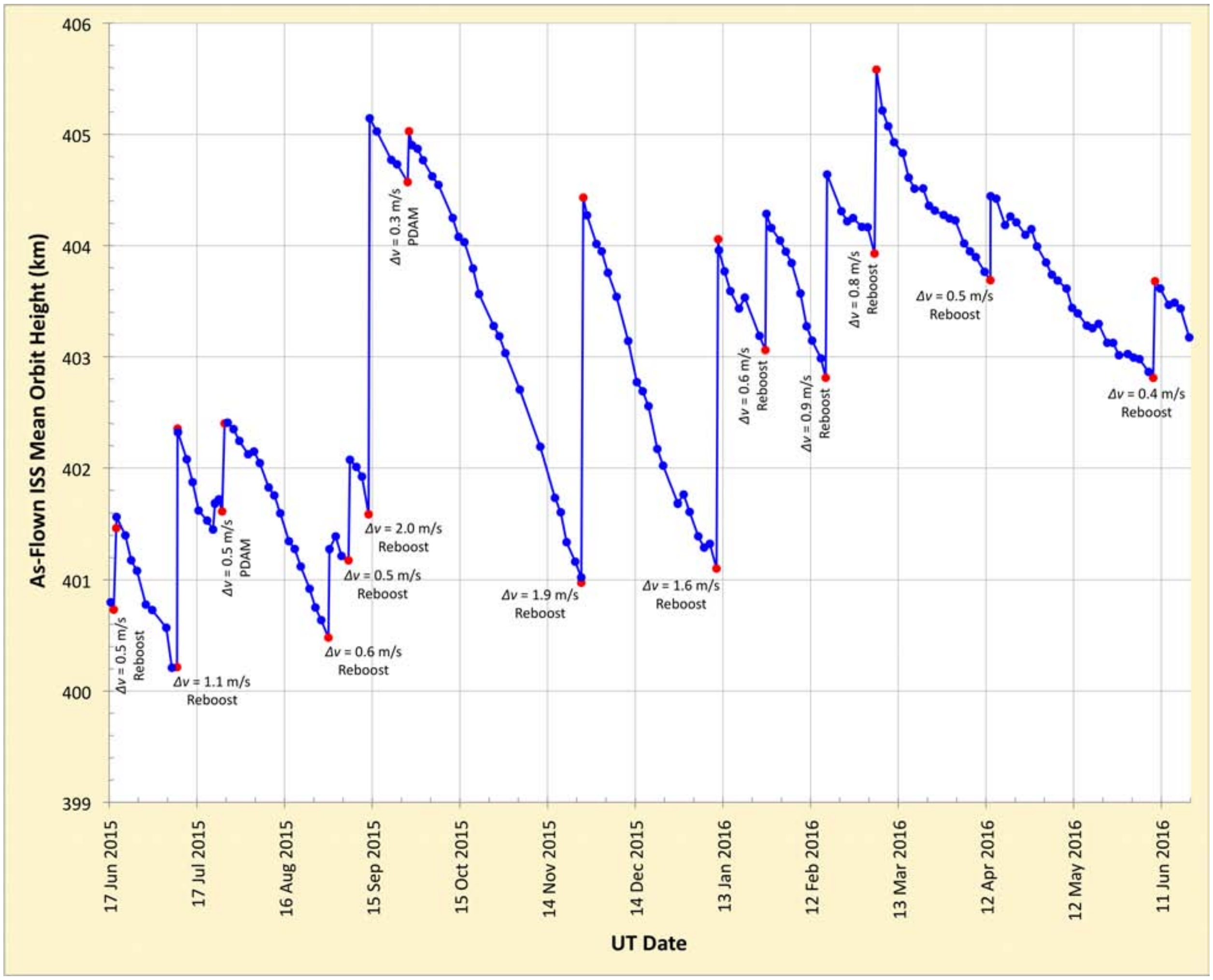

Figure 2. Actual mean orbit height is plotted for ISS during portions of years 2015 and 2016. Orbit reboost impulses are annotated with the associated change-in-velocity magnitude *Δv*. A special type of reboost used to evade possible collisions is called a Pre-determined Debris Avoidance Maneuver (PDAM). Blue data markers are traceable to NASA tracking operations, while red data markers have a USSTRATCOM pedigree.

4. Conclusion

Equations collected in Section 2 are easily programmed into an interactive application or spreadsheet enabling users to readily identify nearly circular orbit heights leading to repeating ground tracks at any orbit inclination of interest. Relevant examples of this process have been included and supported using real world data. The author hopes any mysteries concerning why and how repeating ground tracks arise, whether by human intent or natural coincidence, will be dispelled by this paper's content.

References

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