Reducing Cumulative Solar Eclipse Time In Geosynchronous Earth Orbit

1. Introduction

An impressive number of spaced-based solar power (SBSP) concepts have been proposed as a means of providing inexpensive, sustainable, and plentiful input to Earth's electric power grid. Many SBSP concepts collect sunlight in geosynchronous Earth orbit (GEO), partly because generated energy can be transmitted to a single fixed facility on Earth's surface at any time. In the ideal case of a circular equatorial geo*stationary* Earth orbit (GsEO) at geocentric radius r_{GEO} = 42,164.173 km, the ground track is a single point and energy transmission is simplified by pointing in a fixed direction with respect to the nadir and geocentric velocity.

Arguably the most important benefit from SBSP's venue is highly available sunlight attenuated only by heliocentric distance. Unfortunately, those advocating solar energy collection in GEO often take this virtue too far. For example, an otherwise lucid paper on SBSP in GEO states, "the sun is always shining in space and is not subjected to Earth's seasons" [1, p. 53]. Such statements are, at best, unintentionally misleading because all GEOs experience seasonal solar eclipses by the Earth.

In a geocentric inertial coordinate system, the Sun's annual motion defines a great circle inclined by $\varepsilon = 23.45°$ to Earth's equator [2] and called the ecliptic. The ecliptic can take on a variety of inclinations with respect to a GEO plane, but that plane also defines a geocentric great circle with a line of nodes common to it and the ecliptic. This paper assumes GEOs are of low eccentricity such that their planes remain reasonably fixed in inertial space over a year. Consequently, the Sun's latitude β with respect to a GEO plane will be near zero twice each year. When $\beta = \pm 8.7°$, an observer near r_{GEO} will see the Sun's center tangentially graze Earth's limb around nadir local midnight.* This paper defines the duration of such solar eclipses to be *umbral*, the period when the Sun's entire disc is behind the Earth and received solar power is virtually zero. Since β is measured to the Sun's center, the threshold value at which an instantaneous umbral solar eclipse is possible must be reduced in magnitude by the apparent radius of the Sun (at most 0.3° [2]) to ±8.4°.

In the following section, eclipse durations from an idealized GsEO will be presented during a season with $|\beta| < 8.4°$ to illustrate the magnitude of SBSP outages from that venue. Successive sections will explore how to significantly reduce eclipse durations with respect to the GsEO baseline by introducing GEO planes with appreciable inclinations to Earth's equator. Geocentric motion in one such GEO will then be modeled and assessed to confirm its promised reductions in solar eclipse durations.

To compute eclipse durations, the Sun and Earth are observed from the orbit under assessment around local midnight as rendered by the open source space simulation *Celestia*.† These observations produce eclipse durations with a precision of ±0.1 minute. Figure 1 illustrates

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* Atmospheric refraction observed near r_{GEO} is considered negligible in the context of solar eclipse events.
† A free download of *Celestia* for Windows, Linux, and Mac OS X platforms may be obtained at http://www.shatters.net/celestia/ (accessed 2 May 2016).

Celestia's rendering of a solar eclipse about to begin as seen from GsEO with a prime meridian nadir.

Figure 1. ***Celestia*** **renders the Sun about to slip completely behind Earth as seen from GsEO with a prime meridian nadir on 2016 September 9 at 23:28:45 UT. The observed start of this specific eclipse is 34 s later.**

2. Solar Eclipses As Observed In A GsEO

Means with which to visualize solar eclipse geometry in any GEO are provided by celestial sphere plots (CSPs). A CSP is similar to a conventional world map with longitude replaced by right ascension in the horizontal direction, and latitude replaced by declination in the vertical direction. Although north is still upward on a CSP, it should be noted east is to the left because the celestial sphere is being viewed from the geocenter looking outward at its *inside*. Consequently, prograde GEO motion is from right to left on a CSP.

Figure 2 is a CSP pertaining to any GsEO because they are all coplanar. The orbit plane is plotted in **blue** and coincides with the celestial equator at zero declination. Bracketing the plane are northern ($\beta = +8.4°$) and southern ($\beta = -8.4°$) solar eclipse limits in **red**. Black circles filled in yellow are midnight UT geocentric solar positions at 10-day intervals defining the ecliptic during portions of calendar years 2016 and 2017. Solar positions are annotated with corresponding calendar dates at 30-day intervals. In GsEO, solar eclipses are only possible when the Sun lies between the $|\beta| < 8.4°$ limit lines. Under Figure 2's geometry, solar eclipse seasons are about 45 days in length.

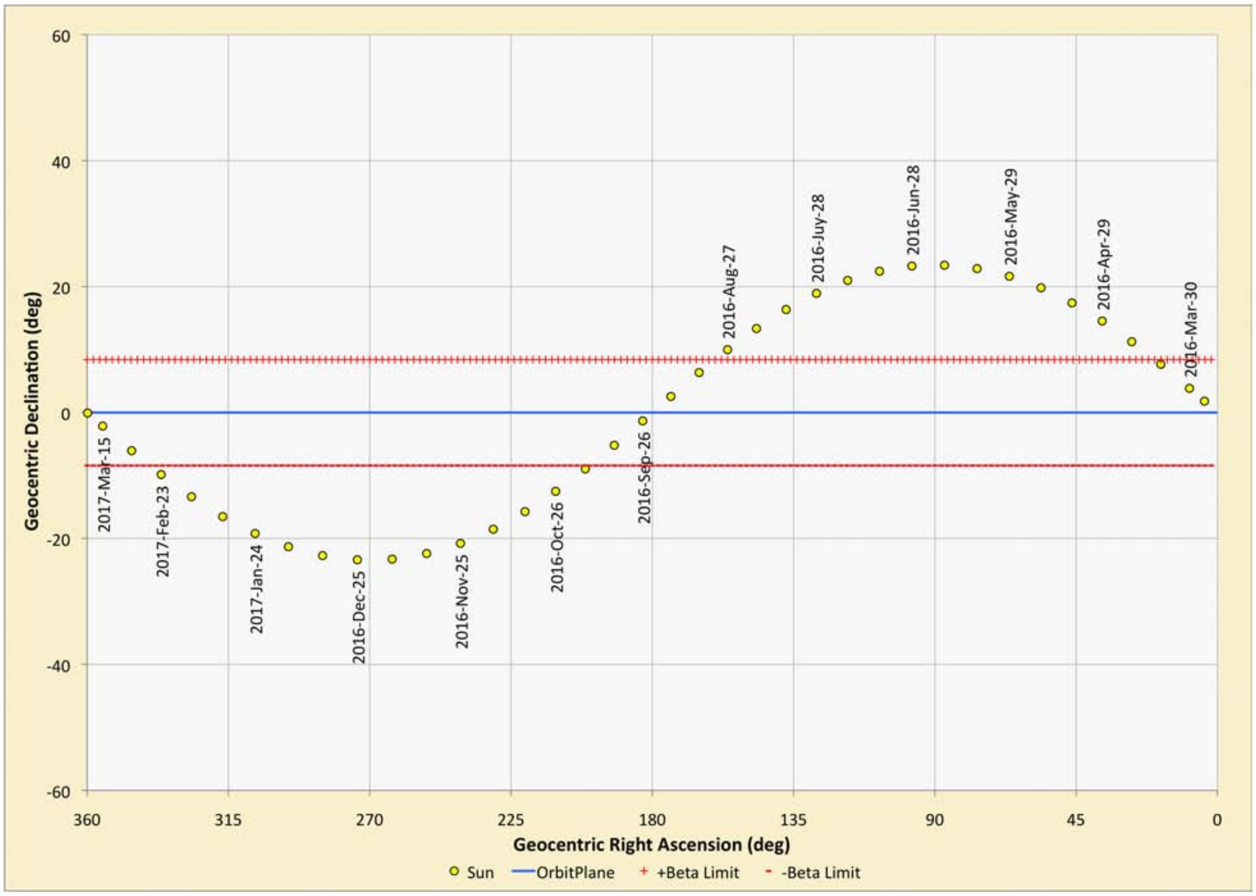

Figure 2. A GsEO plane appears on this CSP in blue along with Sun positions (yellow-filled black circles) at 10-day intervals defining the ecliptic. Solar eclipses will occur in the GsEO only on dates when the Sun has $|\beta| < 8.4°$ and lies between the two red limit lines flanking the GsEO plane.

Visual inspection of Figure 2 indicates a solar eclipse season in GsEO should begin about 31 August and end about 14 October in 2016. To assess this season's eclipses, *Celestia* is configured to observe Earth from an r_{GEO} geocentric distance with a fixed nadir at zero latitude and zero longitude. The UT at which each eclipse starts is noted, together with the corresponding eclipse duration. Figure 3 presents a plot of the latter as a function of the former and spanning the expected GsEO eclipse season. The 44 eclipses in this season have a cumulative duration of 39.1 hours, with the longest eclipse having 67.7-minute duration on 21-22 September UT. The autumnal equinox in 2016 falls on 22 September at 14:20:53 UT as reckoned from the geocenter [2].

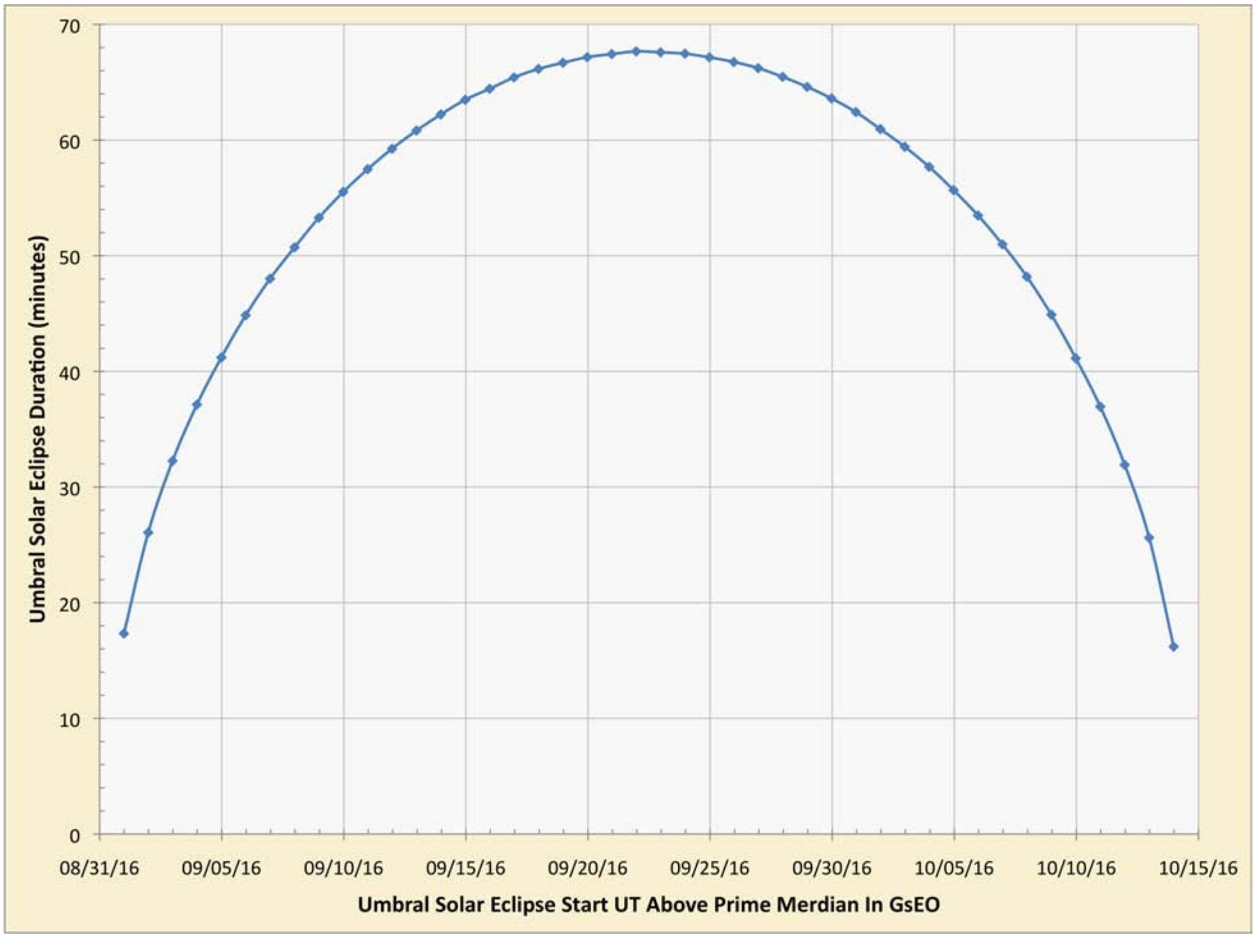

Figure 3. Durations of 44 daily solar eclipses observed from GsEO above the prime meridian near the 2016 autumnal equinox are plotted as a function of eclipse start UT.

3. A Strategy To Reduce Cumulative Solar Eclipse Time In GEO

Additional inspection of Figure 2's CSP indicates cumulative solar eclipse durations during a season in GsEO are neither minimal nor maximal among all GEOs. To reduce cumulative eclipse time with respect to the GsEO baseline, a GEO plane more normal to the ecliptic is required. This geometry is best achieved with a GEO ascending node right ascension Ω at the autumnal equinox (180° on Figure 2's horizontal axis) and sufficient inclination i with respect to Earth's equator. For SBSP applications in GEO, the trade between cumulative solar eclipse duration minimization and Earth ground track dynamics will likely result in selecting moderate i values. With $\Omega = 180°$, a GEO plane normal to the ecliptic is obtained at $i = 90° - \varepsilon = 66.55°$, but the resulting ground track dynamics may be excessive for SBSP transmission purposes.

Experimentation indicates a considerable portion of cumulative eclipse duration reduction in an ecliptic-normal GEO plane can be achieved at $\Omega = 180°$ and $i = 20°$. This plane is therefore adopted for assessment here. Earth ground track dynamics during one day in the adopted GEO plane are illustrated in Figure 4. The UT of ascending node passage on Earth's equator in Figure 4 places this node's longitude at zero.

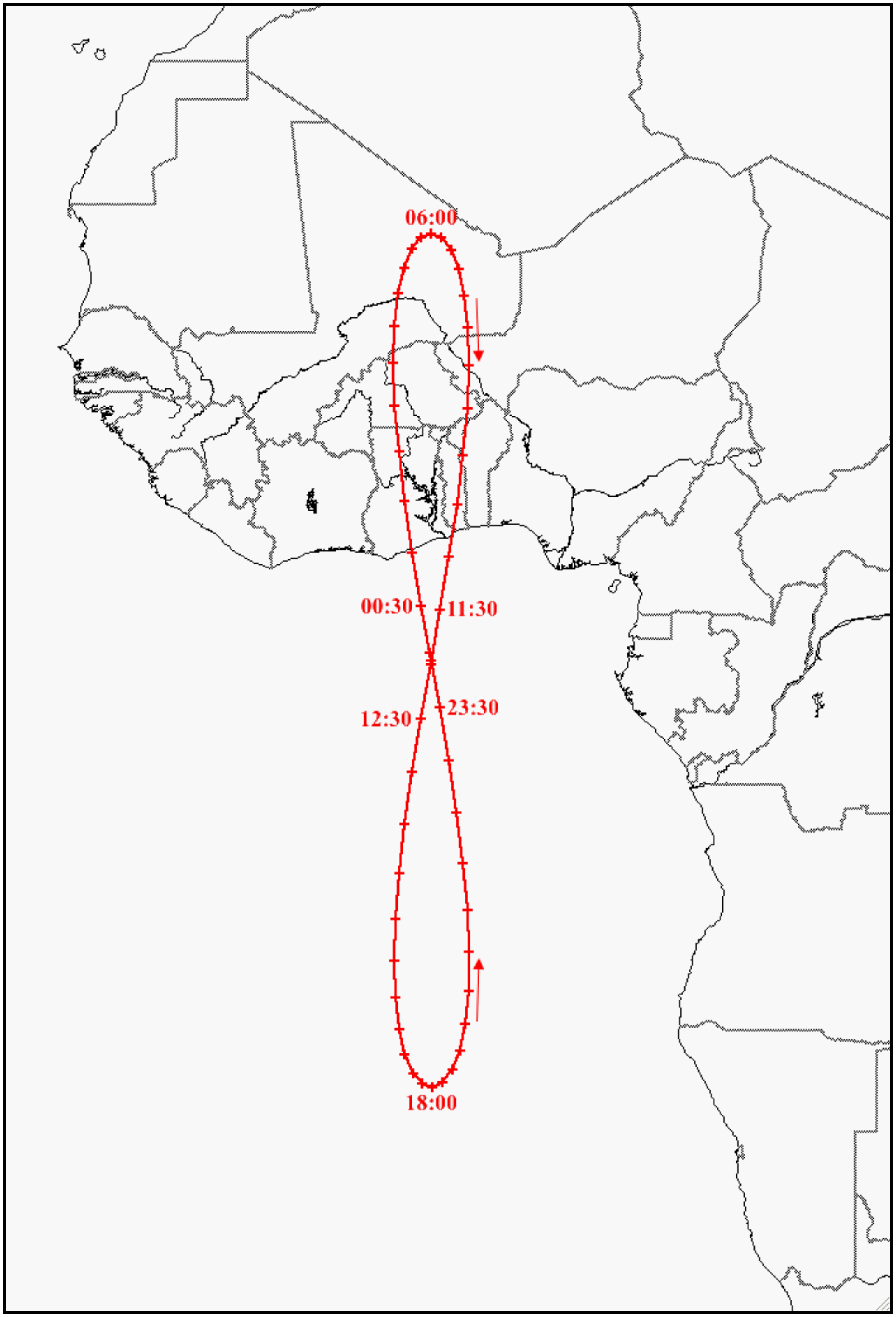

Figure 4. Earth ground track dynamics for a GEO at *i* = 20° with ascending node above the prime meridian are illustrated over one day. Selected ground track "+" markers at 30-minute intervals are annotated with UT in hh:mm format.

Geocentric initial conditions for the $\Omega = 180°$ and $i = 20°$ GEO are specified below for readers wishing to perform their own assessments. This Cartesian state vector's components are with respect to Earth's mean equator and equinox of epoch J2000.0 (J2K).

$$\text{UT} = 20.0 \text{ March } 2016$$

$$\text{J2K position} = \begin{bmatrix} -42131.596 \\ +1655.803 \\ +66.266 \end{bmatrix} \text{km} \qquad \text{J2K velocity} = \begin{bmatrix} -0.111809 \\ -2.887053 \\ +1.051645 \end{bmatrix} \text{km/s}$$

If the plane defined by this state vector were to remain fixed in inertial space, geometry illustrated in Figure 5 would result. Visual inspection of Figure 5 indicates the autumnal equinox solar eclipse season in this GEO should begin about 10 September and end about 5 October in 2016.

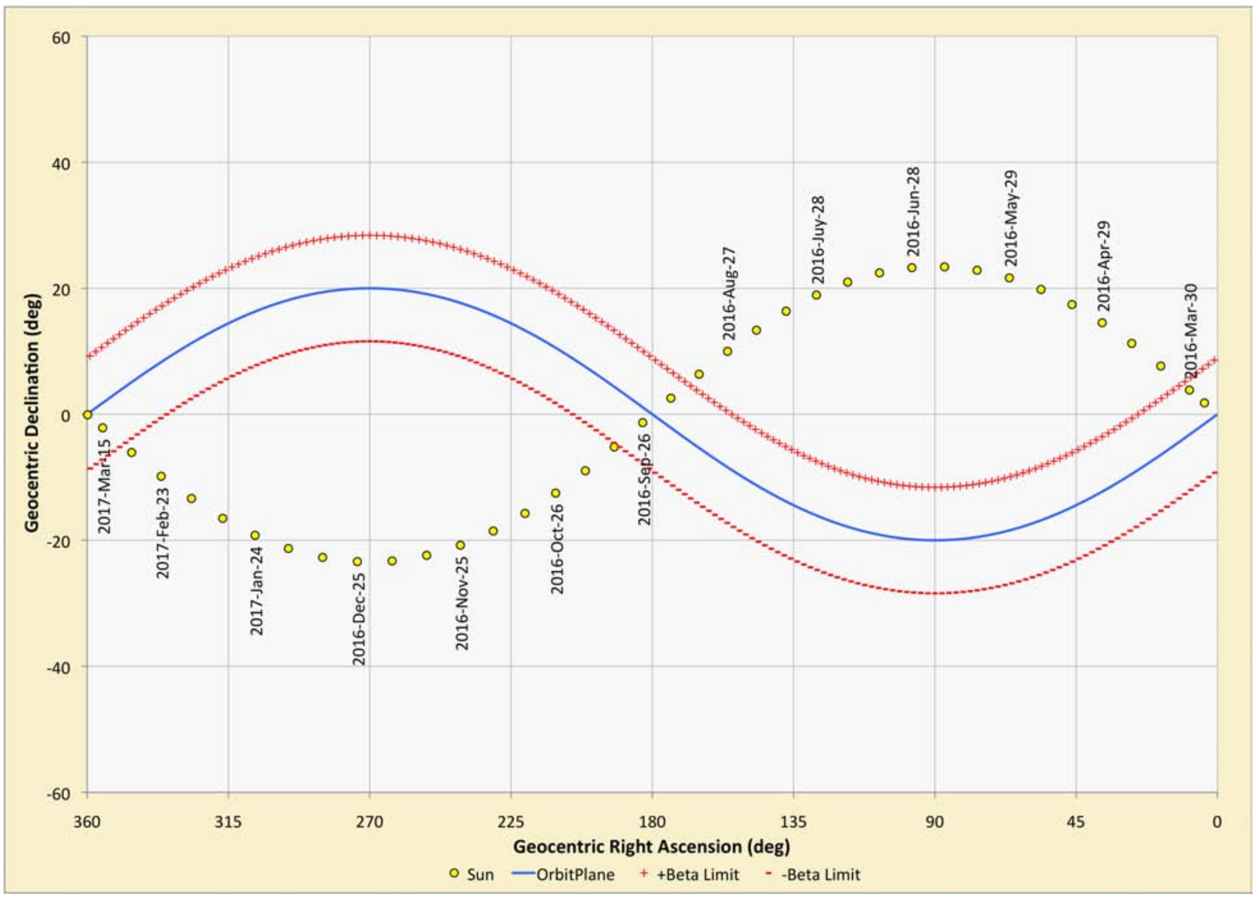

Figure 5. A GEO plane fixed at $\Omega = 180°$ and $i = 20°$ appears on this CSP in blue along with Sun positions (yellow-filled black circles) at 10-day intervals defining the ecliptic. Solar eclipses will occur in the GEO only on dates when the Sun has $|\beta| < 8.4°$ and lies between the two red limit curves flanking the GEO plane.

The WeavEncke orbit predictor [3] coasts the foregoing state vector from its epoch to 1.0 May 2017 UT using a variable integration step size of 30 per orbit (equivalent to a fixed step size near 2872 s for this low-eccentricity GEO). During the coast, accelerations due to point source (Newtonian) gravity from the Earth, Sun, and Moon are modeled along with acceleration from Earth's excess equatorial mass (as scaled by the J_{20} Legendre polynomial coefficient of degree 2 and order zero). The resulting trajectory is exported to *Celestia*, and the 26 solar eclipses observed near the 2016 autumnal equinox from these GEO positions are summarized in Figure 6. These eclipses have a cumulative duration of 22.2 hours, with the longest eclipses having 67.6-minute duration on 18-19 and 19-20 September UT.

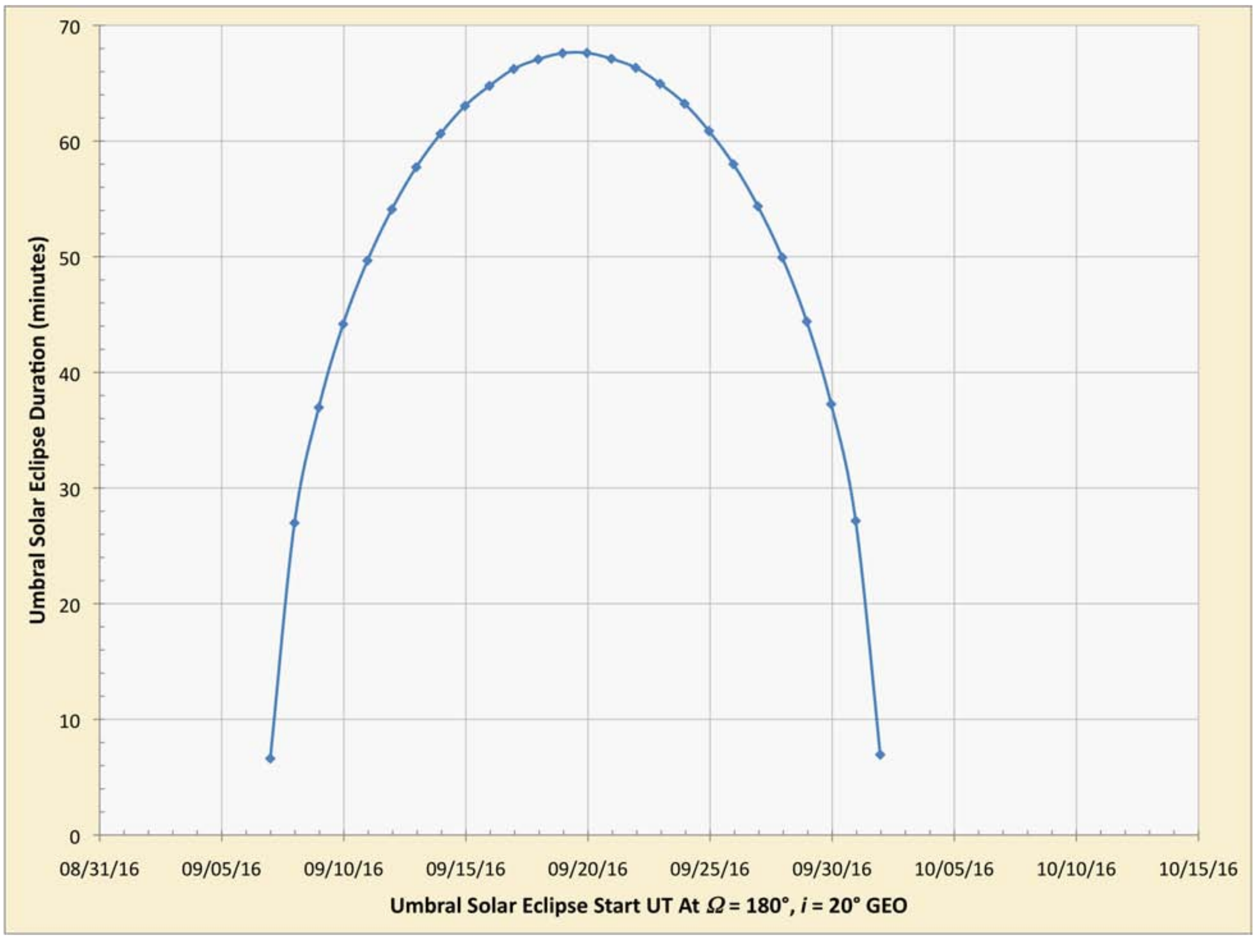

Figure 6. Durations of 26 daily solar eclipses near the 2016 autumnal equinox observed from a GEO with initial Ω = 180° and i = 20° above the prime meridian are plotted as a function of eclipse start UT. Figure 3 axis scales are reproduced here to facilitate comparisons with the GsEO baseline.

With respect to expectations from Figure 5's CSP, the observed eclipse season documented in Figure 6 opens and closes about 3 days early. The skew in season opening and closing dates is due to the coasted GEO plane in Figure 6 slowly precessing westward in inertial space (rightward in Figure 5) from its initial Ω = 180° and i = 20° values (while the Figure 5 GEO plane does remain fixed). All accelerations other than Earth Newtonian gravity being modeled in the WeavEncke coast work to slowly change the Figure 6 GEO plane's orientation after the coast's initialization at 20.0 March 2016 UT half a year earlier. This change is quantified by computing β with respect to a plane fixed in inertial space at $\Omega = 180°$ and $i = 20°$ along with β referenced to the coasted plane. These two sets of β values are plotted as functions of time in Figure 7.

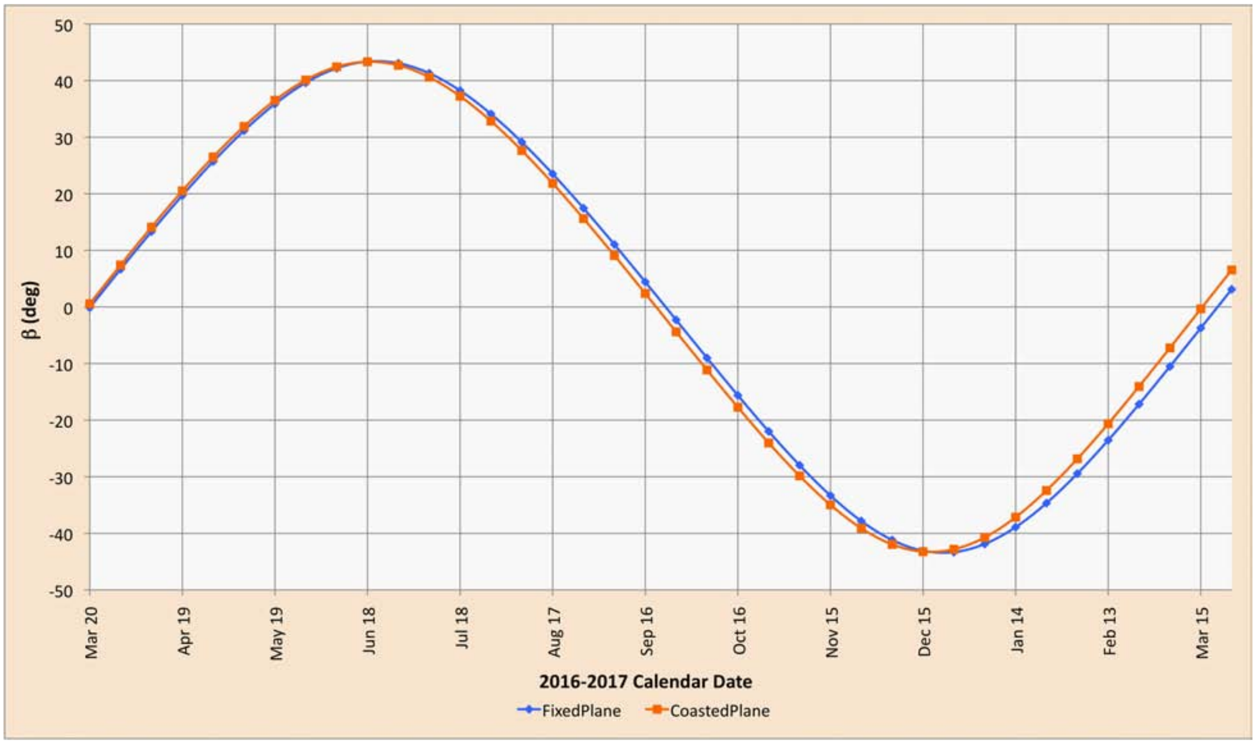

Figure 7. Values for β with respect to a fixed GEO plane at $\Omega = 180°$ and $i = 20°$ are plotted as a function of time (blue) together with β values referenced to the coasted plane whose orientation is perturbed by gravity from the Sun, Moon and Earth's excess equatorial mass (orange). Although the two planes nearly coincide when the coast is initiated on 20.0 March 2016 UT, their β values differ by 2° during the autumnal equinox eclipse season of 2016.

During the 2016 autumnal equinox eclipse season, β is decreasing at the rate of 0.67°/day. At this time, the coasted plane's β is 2° less than that of the fixed plane. Thus, the coasted plane reaches β thresholds opening and closing the eclipse season 2/0.67 = 3 days before the fixed plane.

4. Conclusions

This study of umbral solar eclipses in low-eccentricity GEOs shows them to be unavoidable, contrary to unqualified claims of perpetual sunlight in GEO from some SBSP advocates. In an eclipse, solar power available to onboard systems and remote customers will fall to zero during intervals from minutes to over an hour on a daily basis. Seasons with daily eclipses in GEO extend for weeks without interruption, and cumulative time in Earth's umbra can exceed a day during a single season.

From the limited data presented in this paper, it does not appear maximum solar eclipse duration in a season can be significantly reduced from 68 minutes. This will impose appreciable power storage capacity aboard SBSP infrastructure in GEO. It will also define GEO source-specific power input outage durations to be routinely tolerated by SBSP customer power grids.

But careful selection of the inertial GEO plane can reduce the duration of eclipse seasons, together with cumulative time spent in Earth's umbra during a season. With respect to a GsEO baseline, a simulated GEO initially with $\Omega = 180°$ and $i = 20°$ reduces the number of daily eclipses in a single season from 44 to 26 (a factor of 26/44 = 59%), while cumulative time in Earth's umbra for that season is reduced from 39.1 hours to 22.2 hours (a factor of 22.2/39.1 = 57%). It remains to be demonstrated whether or not ground track dynamics associated with a GEO at $i = 20°$ pose unacceptable hazards or efficiency losses in power transmission from SBSP infrastructure to Earth. Figure 5 also suggests that eclipse seasons can be shifted away from the equinoxes by using GEO planes in orientations such as $\Omega = 90°$ and $i = 20°$. This could lead to constellations of SBSP solar energy collection elements deployed in GEO such that eclipse seasons shift from one to the next in a manageable sequence.

If GEO plane orientation is critical to SBSP operations, means of controlling Ω and i in the presence of perturbations will be necessary. The GEO coast documented in this paper simulates planar drift in inertial space from some relevant perturbations. But it neglects others (such as those from solar radiation pressure) whose accurate simulation requires detailed knowledge of SBSP structure deployed in GEO.

References

[1] Lewis-Weber, J., "Lunar-Based Self-Replicating Solar Factory", *New Space*, Vol. 4, No. 1, 2016, pp. 53-62.‡

[2] Giorgini, J. D., Yeomans, D. K., Chamberlin, A. B., Chodas, P. W., Jacobson, R. A., Keesey, M. S., Lieske, J. H., Ostro, S. J., Standish, E. M., Wimberly, R. N., "JPL's On-Line Solar System Data Service", *Bulletin of the American Astronomical Society*, Vol. 28, No. 3, 1996, p. 1158.§

[3] Adamo, D. R., "A Precision Orbit Predictor Optimized for Complex Trajectory Operations," *Astrodynamics 2003, Advances in the Astronautical Sciences*, Vol. 116, Univelt, San Diego, CA, 2003, pp. 2567-2586.

‡ A copy of this paper may be downloaded at http://online.liebertpub.com/doi/pdfplus/10.1089/space.2015.0041 (accessed 2 May 2016).

§ This service is available at http://ssd.jpl.nasa.gov/?horizons (accessed 2 May 2016).