Introduction

Phobos-Grunt, Russia's sample return mission targeting the martian moon Phobos, was to have marked this nation's return to interplanetary spaceflight after a decades-long hiatus. Launched from Baikonur Cosmodrome, Kazakhstan atop a *Zenit* rocket on 8 November 2011 at 20:16:03 UTC*, *Phobos-Grunt* achieved a nominal Earth parking orbit with apogee/perigee heights of 344/204 km†. The initial Figure 1 ground track terminus is annotated "separation of the SC from the LV". This event occurred 11 min after launch and corresponds to *Phobos-Grunt* separation from the *Zenit* second stage.

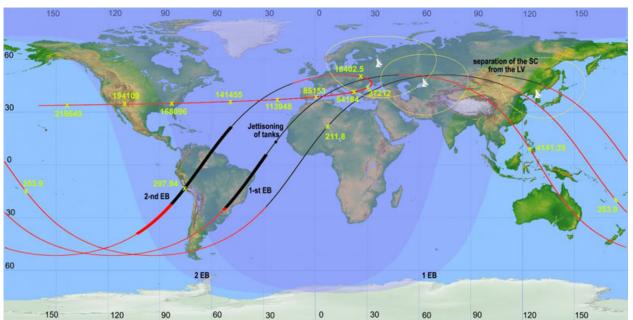


Figure 1. This world map illustrates *Phobos-Grunt*'s planned ground track from Earth parking orbit insertion through two trans-Mars injection (TMI) burns[‡]. The track is colored red when the spacecraft is in sunlight and black when in Earth's shadow. Broader track segments over South America, labeled "1st EB" and "2nd EB", indicate the two TMI burn arcs. Shaded regions, indicating night on Earth's surface during each TMI burn, are labeled "1 EB" and "2 EB" near Antarctica. *Phobos-Grunt* height above Earth in km is annotated in yellow-green with "x" ground track markers. By the time *Phobos-Grunt*'s planned trajectory is over Texas post-TMI, the departing spacecraft was to have been about half the Moon's distance from Earth. Image credit: RussianSpaceWeb.com.

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^{*} This actual launch time and other as-flown events, together with all *Phobos-Grunt* mission planning and performance specifications cited in this article, are obtained from RussianSpaceWeb.com unless noted otherwise.

† These apsis heights are inferred from USSTRATCOM 2-line element set (TLE) #4 with epoch 9 November 2011 at 09:33:24 UTC.

[‡] Throughout this article, a "burn" event refers to planned operation of *Phobos-Grunt* propulsion systems. This is distinct from an "impulse", which refers to an instantaneous approximation of how one or more burns would affect the *Phobos-Grunt* trajectory.

After separation from *Zenit*, *Phobos-Grunt* was to have performed a 2-burn TMI to depart Earth orbit and intercept Mars in September 2012. Both TMI burns, together with the initial Mars orbit insertion (MOI) burn, rely on a modified *Fregat-MT* upper stage known as *Flagman* for propulsion. *Flagman* uses hypergolic propellant and is equipped with drop tanks dedicated to the first TMI burn. After depletion, these tanks are to be left in Earth orbit after the first TMI burn has raised apogee to 4100 km. This event is marked by Figure 1's "Jettisoning of tanks" annotation off the West African coast.

Although telemetry was received from *Phobos-Grunt* as it passed over Russia an orbit after launch, no transmissions from the spacecraft were detected an orbit later. Tracking in this timeframe also confirmed the first TMI burn had not occurred. Some relatively minor propulsive events could be associated with *Phobos-Grunt* tracking in the interval from 10 to 20 November 2011, but nothing resembling the first TMI burn ever occurred. Meanwhile, limited telemetry was received from the spacecraft over Australia on 22 and 23 November 2011, but no capability to reliably command *Phobos-Grunt* was ever established after launch. The original parking orbit ultimately decayed on 15 January 2012.

This article will make no attempt to explain why *Phobos-Grunt* systems were unable to perform TMI. Rather, the intent here is to first estimate the change-in-velocity capability (Δv_C) of *Flagman*. With this Δv_C budget, nominal *Phobos-Grunt* launch season closure is estimated. Finally, the Earth parking orbit into which *Phobos-Grunt* actually launched is assessed to estimate the latest possible date on which the planned mission could be recovered. Whereas the launch season is reported to have closed on 20 November 2011, this season was largely irrelevant to mission recovery after 8 November's actual launch. Following this launch, a "no later than" TMI countdown clock was set to expire in only a few days as *Phobos-Grunt's* Earth parking orbit plane failed to remain adequately aligned with the required Earth departure asymptote bound for Mars.

Consequently, study of hypothetical *Phobos-Grunt* mission recovery scenarios is highly relevant to any interplanetary transportation architecture requiring a multi-launch campaign prepositioning mass in low Earth orbit (LEO) prior to its departure for an interplanetary destination. The first launch in such a campaign also initiates an Earth departure countdown clock that cannot be slipped later by more than a few days.

Estimated Flagman TMI/MOI Capability

Total *Flagman* change-in-velocity capability for *Phobos-Grunt* is defined as the sum of two components such that $\Delta v_C = \Delta v_1 + \Delta v_{23}$. The first component, Δv_I , is generated with propellant from *Flagman*'s drop tanks and applies exclusively to TMI's first burn. After drop tank jettison, Δv_{23} capability is applicable to both the second TMI burn and initial MOI. Throughout Δv_C estimation, a best-case simplifying assumption is made that all *Flagman* burns are applied impulsively to maximize Δv_C . This reinforces the "latest possible" pedigree associated with launch season closure and last possible mission recovery estimates presented subsequently.

Data relevant to Δv_C estimation are as follows.

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m_{il} = total spacecraft mass at Zenit separation and at first TMI burn ignition = 13,500 kg m_{sl} = depleted Flagman drop tanks mass at jettison = 335 kg m_{pl} = usable propellant mass in Flagman drop tanks = 3050 kg m_{p23} = usable propellant mass in Flagman (not including m_{pl}) = 7050 kg I_{SP} = Flagman hypergolic propulsion specific impulse = 333.2 s g = gravitational acceleration at Earth's surface = 0.00980665 km/s<sup>2</sup> v_X = Flagman hypergolic propulsion exhaust speed = gI_{SP} = 3.268 km/s
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The rocket equation then determines both Δv_C components.

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\Delta v_I = v_X \operatorname{Ln}\{m_{il} / (m_{il} - m_{pl})\} = 0.837 \text{ km/s}

\Delta v_{23} = v_X \operatorname{Ln}\{(m_{il} - m_{pl} - m_{sl}) / (m_{il} - m_{pl} - m_{sl} - m_{p23})\} = 3.902 \text{ km/s}
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Summing these components produces $\Delta v_C = 4.739$ km/s.

Estimated Phobos-Grunt Launch Season Closure Date

It is essential to recognize the total change-in-velocity requirement Δv_R associated with any *Phobos-Grunt* launch season date assumes no launch has taken place until that date. This requirement is the sum of two components such that $\Delta v_R = \Delta v_{TMI} + \Delta v_{MOI}$. Because these components are assumed to be instantaneous, the first is computed as a single impulse even though TMI is planned with 2 burns. Since launch on a previous date has not imposed any geometric Earth departure constraints, Δv_{TMI} is assumed perfectly posigrade. Likewise, Δv_{MOI} is assumed perfectly retrograde, rendering Δv_R free of all radial and planar steering losses.

A heliocentric elliptic transfer arc connecting Earth and Mars is fundamental to computing Δv_R . Heliocentric velocities at the termini of this arc are byproducts of a corresponding Lambert boundary value problem solution[§]. Earth-centered speed at the arc's departure terminus is $v_{\infty D}$, and Mars-centered speed at the arc's arrival terminus is $v_{\infty A}$.

At TMI, *Phobos-Grunt* is assumed to be moving in a circular orbit of height $H_{TMI} = 274$ km. This value is the average of apsis heights previously given for *Phobos-Grunt*'s actual Earth parking orbit on 8 November 2011 at 20:16:03 UTC. With the following data**,

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\mu_E = \text{Earth's reduced mass} = 398,600.44 \text{ km}^3/\text{s}^2
R_E = \text{Earth's radius} = 6378.136 \text{ km}
r_{TMI} = R_E + H_{TMI} = 6652.136 \text{ km}
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patched conic theory leads to an expression for Δv_{TMI} .

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[§] Additional *Phobos-Grunt* mission planning information, together with a little experimentation, reveal the launch season of interest utilizes Type II (long-way) Lambert boundary conditions with heliocentric transfer arcs between 180° and 360°

^{**} Physical values for the Earth and Mars provided in this article are obtained from the Jet Propulsion Laboratory's *Horizons* on-line solar system data and ephemeris computation service at http://ssd.jpl.nasa.gov/?horizons.

$$\Delta v_{\scriptscriptstyle TMI} = \sqrt{\frac{2 \mu_{\scriptscriptstyle E}}{r_{\scriptscriptstyle TMI}} + v_{\scriptscriptstyle \infty D}^2} - \sqrt{\frac{\mu_{\scriptscriptstyle E}}{r_{\scriptscriptstyle TMI}}}$$

Following initial MOI, *Phobos-Grunt* mission planning calls for the spacecraft to be at periapsis of a Mars-centered elliptic orbit whose apsis heights are $H_A = 80,000$ km and $H_{MOI} = 800$ km. With the following data,

$$\mu_M = \text{Mars's reduced mass} = 42,828.3 \text{ km}^3/\text{s}^2$$
 $R_M = \text{Mars's radius} = 3394 \text{ km}$
 $r_{MOI} = R_M + H_{MOI} = 4194 \text{ km}$
 $a_{MOI} = R_M + (H_4 + H_{MOI}) / 2 = 43,794 \text{ km}$

patched conic theory leads to an expression for Δv_{MOI} .

$$\Delta v_{MOI} = \sqrt{\frac{2 \mu_{M}}{r_{MOI}} + v_{\infty A}^{2}} - \sqrt{\mu_{M} \left(\frac{2}{r_{MOI}} - \frac{1}{a_{MOI}}\right)}$$

In practice, a set of Lambert solutions is generated for each launch/TMI/departure date in the *Phobos-Grunt* season, beginning with 9 November 2011. While solutions in a set share the same Earth departure date and other Lambert boundary conditions, each Mars arrival date is unique. The solution whose Mars arrival date results in the smallest Δv_R for the set is assessed to determine whether or not the minimal Δv_R is less than Δv_C . The latest launch date on which minimal $\Delta v_R < \Delta v_C$ is the estimated launch season closure date. Figure 2 summarizes results from this analysis.

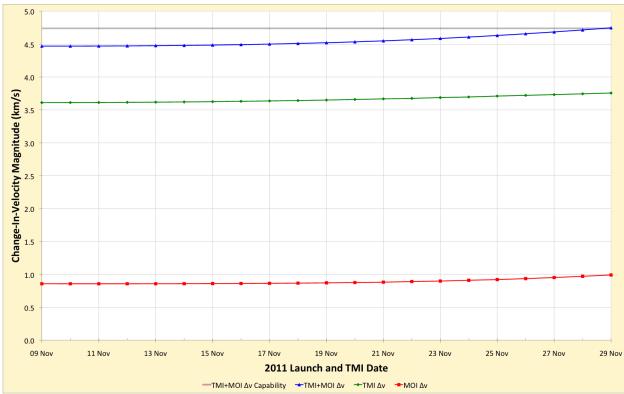


Figure 2. The blue curve in this plot chronicles growth in Δv_R as *Phobos-Grunt* launch date is delayed from its actual occurrence on 8 November 2011. On 29 November 2011, the $\Delta v_R = \Delta v_{TMI} + \Delta v_{MOI}$ curve first exceeds the Δv_C limit plotted in gray. Estimated launch season closure is therefore 28 November 2011. For reference, the green curve plots growth in Δv_{TMI} , and the red curve plots growth in Δv_{MOI} .

The estimated 28 November 2011 launch season closure date inferred from Figure 2 data is 8 days later than that previously cited from a RussianSpaceWeb.com report. This deviation may be due to intentionally optimistic assumptions associated with Figure 2 data. However, Roscosmos head Vladimir Popovkin is quoted as stating on 14 November 2011 that *Phobos-Grunt*'s window for Mars departure would close in early December^{††}. The 28 November 2011 launch season closure estimate may therefore be considered "in the ballpark", particularly if Δv_{MOI} can be reduced by techniques such as increasing H_A .

But the entire discussion of *Phobos-Grunt* launch season closure is academic, if not intentionally misleading, in the context of actual launch having occurred on 8 November 2011. As will be demonstrated in the next two sections, that launch imposes a latest mission recovery date well before even 20 November 2011.

^{††} Reference Emily Lakdawalla's blog at http://www.planetary.org/blog/article/00003261/.

Estimated Single-Impulse TMI Latest Mission Recovery Date

The total change-in-velocity requirement $\Delta v_R'$ associated with *Phobos-Grunt* mission recovery following actual launch on 8 November 2011 is the sum of two components such that $\Delta v_R' = \Delta v_{TMI}' + \Delta v_{MOI}$. For a specified TMI date initiating mission recovery, the Δv_{MOI} component is identical to that required by nominal mission prelaunch planning for that date. But the $\Delta v_{TMI}'$ component will generally require steering through the angle β in order to turn the geocentric *Phobos-Grunt* Earth parking orbit plane into one containing the required Earth departure asymptote bound for Mars. Assuming this steering is done simultaneously with the TMI geocentric speed increase (the most propellant-conservative single-impulse strategy), associated vector geometry is illustrated in Figure 3.

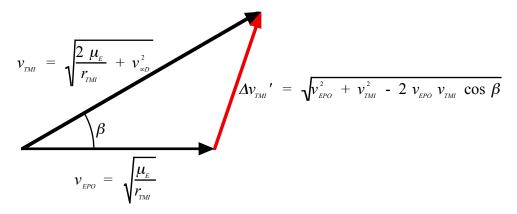


Figure 3. This geocentric velocity vector diagram forms a triangle with sides whose lengths are geocentric speeds. The first side (smaller black arrow) has speed in Earth parking orbit v_{EPO} before TMI. The second side (larger black arrow) has speed in the Earth departure hyperbola v_{TMI} immediately after TMI. The third side (red arrow) has the change-in-velocity magnitude Δv_{TMI} associated with TMI as computed by the law of cosines. When the steering angle β is zero, Δv_{TMI} is simply v_{TMI} minus v_{EPO} , as previously computed for a nominal mission's Δv_{TMI} .

Computing β is not a trivial process. Regular USSTRATCOM updates to the as-flown *Phobos-Grunt* trajectory in its Earth parking orbit are processed to determine the spacecraft's angular momentum vector c in geocentric inertial space. Although c is normal to *Phobos-Grunt*'s orbit plane at any instant, excess mass about Earth's equator causes c to precess westward at about 5.4° per day. Meanwhile, asymptotic Earth departure velocity $v_{\infty D}$ is slowly changing with time in geocentric inertial space due to Earth and Mars heliocentric motion. Because it measures the angle between a vector and a plane, β is equivalent to $v_{\infty D}$ latitude with respect to the *Phobos-Grunt* Earth parking orbit plane at a specified mission recovery TMI time. Adopting the sign convention " β is positive when $v_{\infty D}$ points into the hemisphere whose pole is c", the following equation computes its value**.

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^{‡‡} A similar quantity called " β " is routinely used in planning International Space Station (ISS) operations, and it has the same sign convention. Of course, this parameter defines c with respect to ISS orbit elements. The only fundamental difference is the ISS β context replaces $v_{\infty D}$ with the Sun's geocentric position vector.

$$\beta = 90^{\circ} - a\cos\left\{\frac{c \cdot v_{\infty}}{c v_{\infty}}\right\}$$

The foregoing computational pedigree applies to hypothetical *Phobos-Grunt* mission recovery data summarized in Table 1. From these data, it is evident a single-impulse TMI *Phobos-Grunt* mission recovery option existed for little more than 3 days after actual launch.

Table 1. Hypothetical *Flagman* propulsion requirement $\Delta v_R'$ for single-impulse TMI *Phobos-Grunt* mission recovery is assessed on TMI dates following actual launch on 8 November 2011. Values for Δv_{TMI} are included to compare with corresponding $\Delta v_{TMI}'$ values because the former assume steering angle β is zero. Since increasing β rapidly inflates $\Delta v_{TMI}'$ as mission recovery TMI is postponed, $\Delta v_R'$ exceeds *Flagman* propulsive capability $\Delta v_C = 4.739$ km/s before 12 November 2011.

Parameter	2011 Date at Hypothetical Mission Recovery TMI						
1 at atticues	9 Nov	10 Nov	11 Nov	12 Nov			
2012 Mars Arrival	11 Sep	11 Sep	12 Sep	12 Sep			
β (deg)	+0.260	+3.790	+7.503	+11.204			
Δv_{TMI} (km/s)	3.611	3.612	3.613	3.615			
$\Delta v_{TMI}'$ (km/s)	3.611	3.665	3.816	4.052			
$\Delta v_{MOI} (\text{km/s})$	0.858	0.858	0.857	0.857			
$\Delta v_R'$ (km/s)	4.469	4.523	4.673	4.909			

Inertial dynamics giving rise to β variations can be visualized by projecting snapshots of the precessing *Phobos-Grunt* Earth parking orbit plane onto the geocentric celestial sphere, along with variations in the direction of $v_{\infty D}$. Like the Figure 1 Earth map, north is up and south is down in the Figure 4 celestial sphere plot. In this analogy, Figure 1 latitude is replaced by Figure 4 declination with respect to the Earth mean equator of Julian epoch J2000.0. Likewise, Figure 1 longitude is replaced by right ascension with respect to the mean equinox at J2000.0 in Figure 4. Because Figure 4 shows the inside of a celestial sphere rather than Earth's surface, east is left and west is right. Consequently, the *Phobos-Grunt* orbit plane drifts rightward with time in Figure 4.

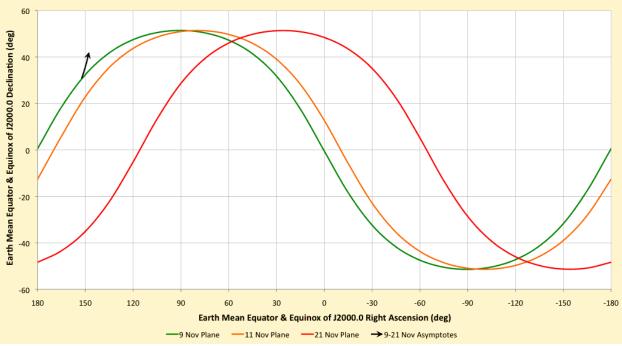


Figure 4. Snapshots of the actual *Phobos-Grunt* Earth parking orbit plane on 9 November 2011 (green), 11 November 2011 (orange), and 21 November 2011 (red) are projected onto the geocentric celestial sphere (truncated at declination magnitudes exceeding 60°) defined by Earth's mean equator and equinox of Julian epoch J2000.0. These snapshots illustrate westward precession of the plane with time. In addition, the slowly changing Earth asymptotic departure direction for Mars is plotted as a black arrow with the 9 November 2011 departure direction at the "tail" end and the 21 November 2011 departure direction at the "head" end. The asymptotic departure locus lies closest to the *Phobos-Grunt* orbit plane on 9 November 2011, shortly after actual launch. Only then are TMI propulsive steering losses due to increasing β negligible. By 11 November 2011, these losses are about to exceed estimated *Flagman* capability to recover the mission with a single TMI impulse.

At the 5.4° per day precession rate, about 5 additional weeks would be required for the 21 November 2011 plane to precess near the plotted asymptotic departure locus in Figure 4. By that time in late December 2011, Earth will have phased too close to Mars opposition for *Phobos-Grunt* mission recovery with *Flagman* propulsive capability. Note also the possibility that asymptotic departure declination can drift so far north (or south) that it exceeds the orbit plane's northern (or southern) declination limit. Under such geometry, TMI β might never be sufficiently near zero, regardless of orbit plane precession in right ascension.

Estimated Three-Impulse TMI Latest Mission Recovery Date

Now assume *Phobos-Grunt* could have performed TMI with three impulses. The first impulse would actually be performed with two *Flagman* burns, the first burn depleting its drop tanks. A four-burn *Flagman* TMI capability enables a strategy whose total change-in-velocity requirement is Δv_R ". Under this strategy, *Phobos-Grunt* mission recovery following actual launch on 8 November 2011 is the sum of four components such that Δv_R " = $\Delta v_{HA} + \Delta v_{NPC} + \Delta v_{TMI}$ " + Δv_{MOI} .

The first three components of Δv_R " comprise TMI in the foregoing expression, while the Δv_{MOI} component is identical to that required by nominal mission prelaunch planning for launch on the recovery date.

The three-impulse TMI takes maximum advantage of two propellant-conserving precepts in astrodynamics. First, a change in speed is best performed at the fastest possible initial speed. In the TMI context, this entails performing posigrade impulses at perigee. Second, a change in direction is best performed at the slowest possible speed, dictating β be reduced to zero at apogee.

In accord with these precepts, the first "height adjust" impulse Δv_{HA} is posigrade and raises the assumed initial Earth parking orbit's circular height at 274 km to some minimal apogee radius r_A . Since there is no distinct perigee in the initial orbit, the Δv_{HA} impulse establishes a perigee consistent with the required Earth departure hyperbola. The second "plane change" impulse Δv_{NPC} is performed at r_A and achieves $\beta = 0$ at the next perigee without any change in speed. The third Δv_{TMI} " impulse is posigrade and performed at that next perigee, whose 274 km height is unaltered from the original parking orbit. Readers familiar with "anytime" lunar return trajectory planning for the Constellation Program will recognize the three-impulse TMI recovery as a fundamentally similar strategy.

As r_A increases with increased Δv_{HA} , the Δv_{NPC} required to null a specified β decreases. The downside to this trend is orbit period increases with r_A , delaying Δv_{TMI} " and causing all three TMI components to increase. Consequently, r_A is increased no more than necessary to reduce Δv_R " within Flagman's $\Delta v_C = 4.739$ km/s.

In assessing the three-impulse TMI, it is important to recognize that the first Δv_{HA} impulse at time t_I will place r_A well above a geostationary orbit's radius, all but halting westward precession of the resulting orbit. Although β will continue to increase after t_I , it will do so only during brief near-perigee intervals immediately after t_I and before Δv_{TMI} " at time t_3 and while the Earth departure asymptote drifts slowly northwestward per Figure 4. In a continuing effort to impart maximum possible *Phobos-Grunt* mission recovery capability, β is therefore frozen at its t_I value during each assessment.

A second consideration when assessing three-impulse TMI is growth in all but the Δv_{NPC} component of $\Delta v_R''$ during the interval from t_1 to t_3 . But $\Delta v_R'' - \Delta v_{NPC} = \Delta v_{HA} + \Delta v_{TMI}'' + \Delta v_{MOI}$ is just another way of computing Δv_R in a context specific to three-impulse TMI. To compute Δv_R at a specified t_3 , a least squares cubic polynomial is fit to all Δv_R points plotted in Figure 2. If t_3 is expressed as a November 2011 UTC decimal day, the following cubic reproduces all Figure 2 Δv_R values in km/s units to at least 0.001 precision.

$$\Delta v_R^* = 0.00001565741 * t_3^3 - 0.00001729571 * t_3^2 - 0.00392343 * t_3 + 4.494745$$

During each three-impulse TMI assessment, the minimum r_A is sought for which $\Delta v_{NPC} < \Delta v_C - \Delta v_R^*$. At any r_A being assessed, t_3 is one orbit period after t_2 . This period is computed assuming

perigee height remains at 274 km. Three-impulse TMI assessment results are presented in Table 2.

Table 2. Hypothetical three-impulse TMI *Phobos-Grunt* mission recovery is assessed for TMI dates following actual launch on 8 November 2011 and loss of single-impulse TMI mission recovery capability three days later. A minimal apogee radius r_A is targeted by the first Δv_{HA} posigrade impulse at time t_1 such that Δv_R " does not exceed the previously computed $\Delta v_C = 4.739$ km/s *Flagman* capability. The second Δv_{NPC} impulse can then null the TMI steering angle β such that the third Δv_{TMI} " impulse at time t_3 is purely posigrade. Values for $\Delta v_R^* = \Delta v_R$ " - Δv_{NPC} are from a polynomial approximation to Δv_R data plotted in Figure 2.

Parameter	2011 UTC at Hypothetical Mission Recovery Δv_{HA} (t_I)							
	12.0 Nov	13.0 Nov	14.0 Nov	15.0 Nov	16.0 Nov	17.0 Nov		
2012 Mars Arrival	12 Sep	13 Sep	13 Sep	14 Sep	14 Sep	15 Sep		
β (deg)	+11.204	+15.048	+18.875	+22.805	+26.706	+30.677		
Δv_R^* (km/s)	4.474	4.479	4.486	4.498	4.515	4.545		
r_A (km)	51,000	71,000	92,000	117,000	147,000	196,000		
Δv_{NPC} (km/s)	0.262	0.257	0.251	0.239	0.224	0.193		
<i>t</i> ₃ (2011 Nov UTC)	12.564	13.881	15.262	16.771	18.453	20.715		
$\Delta v_R'' \text{ (km/s)}$	4.736	4.736	4.737	4.737	4.739	4.738		

Table 2 confirms Flagman's Δv_C constraint imposes a runaway increase in r_A in order to decrease Δv_{NPC} as β and Δv_R^* increase with postponed t_1 and t_3 , respectively. To be a viable mission recovery option, a three-impulse TMI must be initiated such that t_1 is earlier then 18.0 November 2011 UTC. By that time, r_A has grown to the point t_3 falls after 29 November 2011. As noted in Figure 2's caption, a t_3 this late drives Δv_R to exceed Δv_C , and no Flagman capability is available to perform Δv_{NPC} .

Conclusion

The *Phobos-Grunt* mission's failure to achieve TMI serves as an empirical demonstration of the difference between a launch season and the interval in which a mission may be recovered after an otherwise nominal launch into LEO leads to delayed departure for deep space. An inexorable mission recovery countdown clock is running during the delay. In the *Phobos-Grunt* case, this clock expired 3 to 9 days after actual launch, depending on the number of TMI burns mission managers were willing to perform. This estimated mission recovery interval is but a fraction of the mission's 20-day launch season, even if the season is assumed to have opened on the day *Phobos-Grunt* launched. This situation was never clearly communicated as it played out in November 2011.

But there are broader implications from the *Phobos-Grunt* mission recovery scenario. A similar countdown clock is set following the first of multiple launches required to build up sufficient mass in LEO for departure to any interplanetary destination. An adequately padded launch

campaign timeline manages the risk of late departure, but additional exposure to the LEO environment carries its own risks.

Reusable infrastructure in LEO, a propellant depot being a notable example, will be particularly challenged to ensure β is sufficiently near zero at a time when an interplanetary destination is properly phased with Earth. This may require deploying such reusable infrastructure at a sufficiently high orbit inclination to guarantee all conceivable Earth departure asymptote declinations are accommodated. Sufficiently high inclination will generally incur a performance penalty for all launches supporting the reusable infrastructure's logistics. It may therefore be preferable to adopt single-use, mission-specific architectures for multiple-launch interplanetary mission campaigns if they must be staged in LEO.