Phobos-Grunt's Inexorable Trans-Mars Injection (TMI) Countdown Clock

Introduction

Phobos-Grunt, Russia's sample return mission targeting the martian moon Phobos, was to have marked this nation's return to interplanetary spaceflight after a decades-long hiatus. Launched from Baikonur Cosmodrome, Kazakhstan atop a *Zenit* rocket on 8 November 2011 at 20:16:03 UTC*, *Phobos-Grunt* achieved a nominal Earth parking orbit with apogee/perigee heights of 344/204 km†. The initial Figure 1 ground track terminus is annotated "separation of the SC from the LV". This event occurred 11 min after launch and corresponds to *Phobos-Grunt* separation from the *Zenit* second stage.

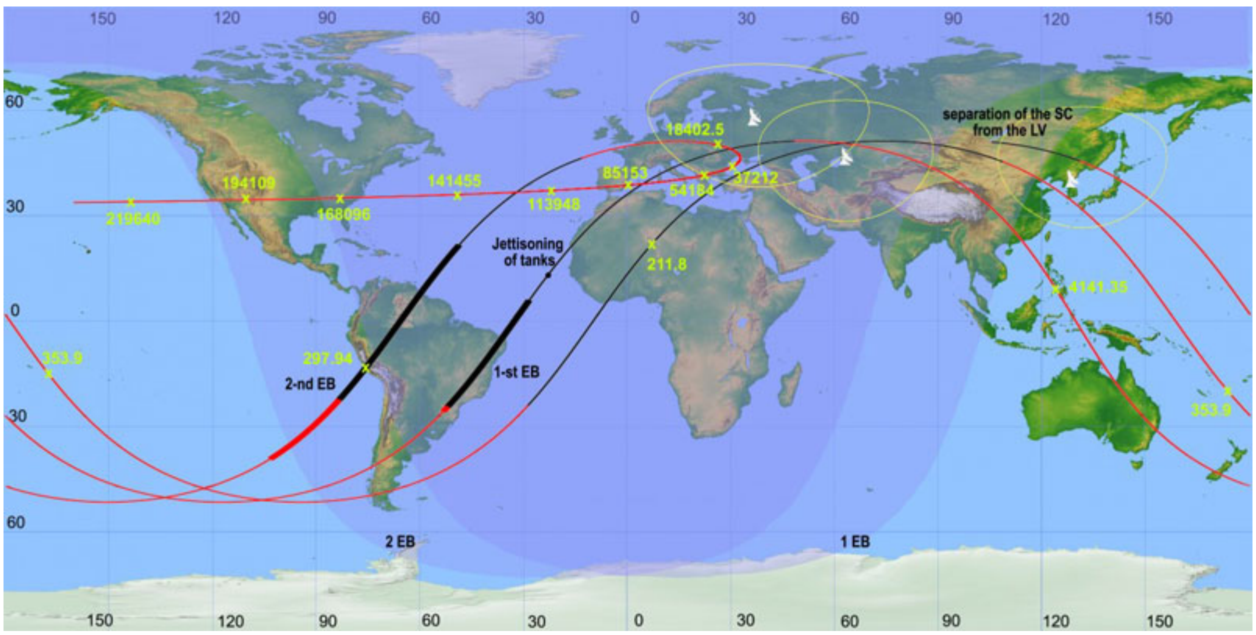

Figure 1. This world map illustrates *Phobos-Grunt*'s planned ground track from Earth parking orbit insertion through two trans-Mars injection (TMI) burns. The track is colored red when the spacecraft is in sunlight and black when in Earth's shadow. Broader track segments over South America, labeled "1st EB" and "2nd EB", indicate the two TMI burn arcs. Shaded regions, indicating night on Earth's surface during each TMI burn, are labeled "1 EB" and "2 EB" near Antarctica. *Phobos-Grunt* Earth orbit height in km is annotated in yellow-green with "x" ground track markers. By the time *Phobos-Grunt*'s planned trajectory is over Texas post-TMI, the departing spacecraft was to have been about half the Moon's distance from Earth. Image credit: RussianSpaceWeb.com.

After separation from *Zenit*, *Phobos-Grunt* was to have performed a 2-stage TMI burn to depart Earth orbit and intercept Mars next summer. Both TMI stages, together with the initial Mars orbit insertion (MOI) burn, rely on a modified *Fregat*-MT upper stage known as *Flagman* for propulsion. *Flagman* uses hypergolic propellant and is equipped with drop tanks dedicated to

* This actual launch time and other as-flown events, together with all *Phobos-Grunt* mission planning and performance specifications cited in this article, are obtained from RussianSpaceWeb.com unless noted otherwise.
† These apsis heights are inferred from USSTRATCOM 2-line element set (TLE) #4 with epoch 9 November 2011 at 09:33:24 UTC.

the first TMI burn. After depletion, these tanks are to be left in Earth orbit after the first TMI burn has raised apogee to 4100 km. This event is marked by Figure 1's "Jettisoning of tanks" annotation off the West African coast.

Although telemetry was received from *Phobos-Grunt* as it passed over Russia an orbit after launch, no transmissions from the spacecraft were detected an orbit later. Tracking in this timeframe also confirmed the first TMI burn had not occurred. Some relatively minor propulsive events could be associated with *Phobos-Grunt* tracking in the interval from 10 to 20 November 2011, but nothing resembling the first TMI burn ever occurred. Meanwhile, limited telemetry was received from the spacecraft over Australia on 22 and 23 November 2011, but no capability to reliably command *Phobos-Grunt* was ever established after launch. The original parking orbit ultimately decayed on TBS January 2012.

This article will make no attempt to explain why *Phobos-Grunt* systems were unable to perform TMI. Rather, the intent here is to first estimate the change-in-velocity capability (Δv_C) of *Flagman*. With this Δv_C budget, nominal *Phobos-Grunt* launch season closure is estimated. Finally, the Earth parking orbit into which *Phobos-Grunt* actually launched is assessed to estimate the latest possible date on which the planned mission could be recovered. Whereas the launch season is reported to have closed on 20 November 2011, this season was irrelevant to mission recovery after 8 November's actual launch. Following this launch, a "no later than" TMI countdown clock was set to expire in only a few days as *Phobos-Grunt's* Earth parking orbit plane failed to adequately align with the required Earth departure asymptote bound for Mars.

Estimated *Flagman* TMI/MOI Capability

Total *Flagman* change-in-velocity capability for *Phobos-Grunt* is defined as the sum of two components such that $\Delta v_C = \Delta v_1 + \Delta v_{23}$. The first component, Δv_1, is generated with propellant from *Flagman*'s drop tanks and applies exclusively to TMI's first stage. After drop tank jettison, Δv_{23} capability is applicable to both second stage TMI and initial MOI. Throughout Δv_C estimation, a best-case simplifying assumption is made that all *Flagman* burns are applied impulsively to maximize Δv_C. This reinforces the "latest possible" pedigree associated with launch season closure and last possible mission recovery estimates presented subsequently.

Data relevant to Δv_C estimation are as follows.

m_{i1} ≡ total spacecraft mass at *Zenit* separation and at TMI first stage ignition = 13,500 kg
m_{s1} ≡ depleted *Flagman* drop tanks mass at jettison = 335 kg
m_{p1} ≡ usable propellant mass in *Flagman* drop tanks = 3050 kg
m_{p23} ≡ usable propellant mass in *Flagman* (not including m_{p1}) = 7050 kg
I_{SP} ≡ *Flagman* hypergolic propulsion specific impulse = 333.2 s
g ≡ gravitational acceleration at Earth's surface = 0.00980665 km/s
v_X ≡ *Flagman* hypergolic propulsion exhaust speed = $g\, I_{SP}$ = 3.268 km/s

The rocket equation then determines both Δv_C components.

$\Delta v_1 = v_X \,\mathrm{Ln}\{m_{i1} / (m_{i1} - m_{p1})\} = 0.837$ km/s
$\Delta v_{23} = v_X \,\mathrm{Ln}\{(m_{i1} - m_{p1} - m_{s1}) / (m_{i1} - m_{p1} - m_{s1} - m_{p23})\} = 3.902$ km/s

Summing these components produces $\Delta v_C = 4.739$ km/s.

Estimated *Phobos-Grunt* Launch Season Closure Date

It is essential to recognize the total change-in-velocity requirement Δv_R associated with any *Phobos-Grunt* launch season date assumes no launch has taken place until that date. This requirement is the sum of two components such that $\Delta v_R = \Delta v_{TMI} + \Delta v_{MOI}$. Because these components are assumed to be impulsive, the first is computed as a single burn even though TMI is planned in 2 stages. Since launch on a previous date has not imposed any geometric Earth departure constraints, Δv_{TMI} is assumed perfectly posigrade. Likewise, Δv_{MOI} is assumed perfectly retrograde, rendering Δv_R free of all radial and planar steering losses.

A heliocentric elliptic transfer arc connecting Earth and Mars is fundamental to computing Δv_R. Heliocentric velocities at the termini of this arc are byproducts of a corresponding Lambert boundary value problem solution‡. Earth-centered speed at the arc's departure terminus is $v_{\infty D}$, and Mars-centered speed at the arc's arrival terminus is $v_{\infty A}$.

At TMI, *Phobos-Grunt* is assumed to be moving in a circular orbit of height $H_{TMI} = 274$ km. This value is the average of apsis heights previously given for *Phobos-Grunt*'s actual Earth parking orbit on 8 November 2011 at 20:16:03 UTC. With the following data§,

$\mu_E \equiv$ Earth's reduced mass = 398,600.44 km^3/s^2
$R_E \equiv$ Earth's radius = 6378.136 km
$r_{TMI} = R_E + H_{TMI} = 6652.136$ km

patched conic theory leads to an expression for Δv_{TMI}.

$$\Delta v_{TMI} = \sqrt{\frac{2\,\mu_E}{r_{TMI}} + v_{\infty D}^2} - \sqrt{\frac{\mu_E}{r_{TMI}}}$$

Following initial MOI, *Phobos-Grunt* mission planning calls for the spacecraft to be at periapsis of a Mars-centered elliptic orbit whose apsis heights are $H_A = 80{,}000$ km and $H_{MOI} = 800$ km. With the following data,

$\mu_M \equiv$ Mars's reduced mass = 42,828.3 km^3/s^2
$R_M \equiv$ Mars's radius = 3394 km

‡ Additional *Phobos-Grunt* mission planning information, together with a little experimentation, reveal the launch season of interest utilizes Type II (long-way) Lambert boundary conditions with heliocentric transfer arcs between 180° and 360°.
§ Physical values for the Earth and Mars provided in this article are obtained from the Jet Propulsion Laboratory's *Horizons* on-line solar system data and ephemeris computation service at http://ssd.jpl.nasa.gov/?horizons.

$r_{MOI} = R_M + H_{MOI} = 4194$ km
$a_{MOI} = R_M + (H_A + H_{MOI}) / 2 = 43{,}794$ km

patched conic theory leads to an expression for Δv_{MOI}.

$$\Delta v_{MOI} = \sqrt{\frac{2\,\mu_M}{r_{MOI}} + v_{\infty A}^2} - \sqrt{\mu_M \left(\frac{2}{r_{MOI}} - \frac{1}{a_{MOI}}\right)}$$

In practice, a set of Lambert solutions is generated for each launch/TMI/departure date in the *Phobos-Grunt* season, beginning with 9 November 2011. While solutions in a set share the same Earth departure date and other Lambert boundary conditions, each Mars arrival date is unique. The solution whose Mars arrival date results in the smallest Δv_R for the set is assessed to determine whether or not the minimal Δv_R is less than Δv_C. The latest launch date on which minimal $\Delta v_R < \Delta v_C$ is the estimated launch season closure date. Figure 2 summarizes results from this analysis.

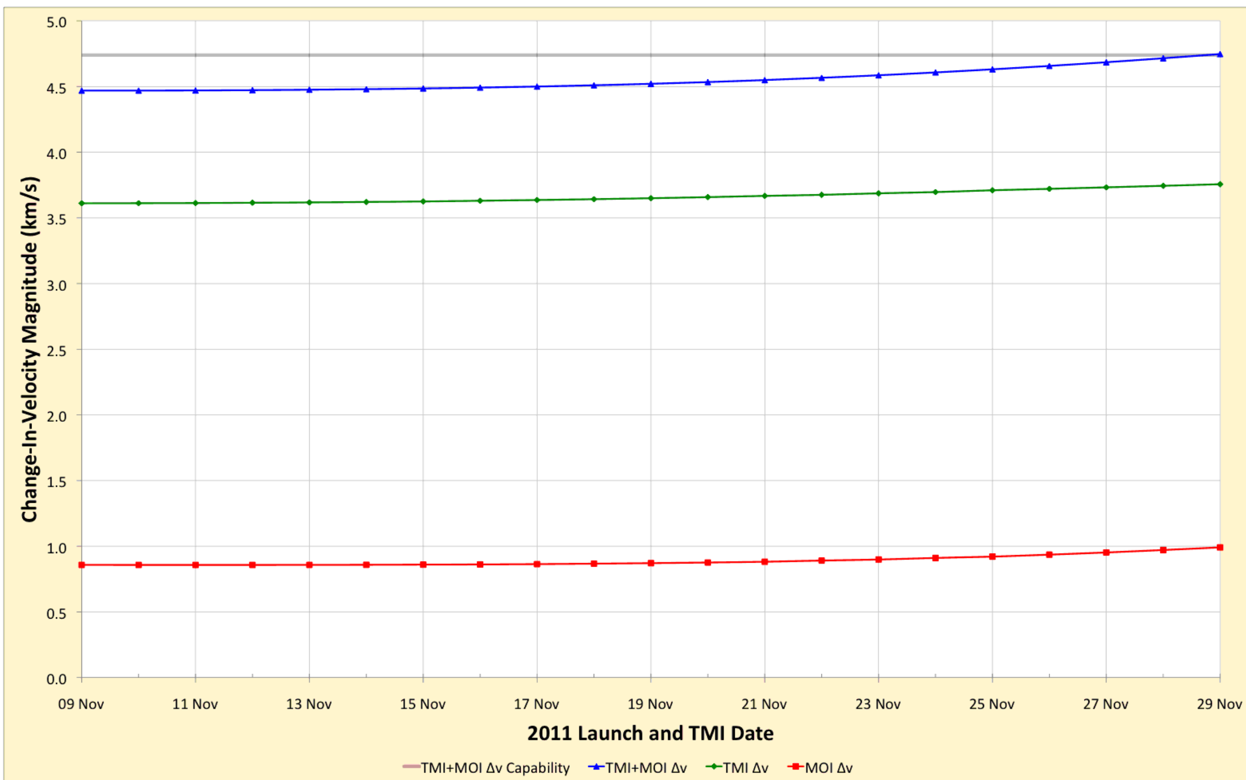

Figure 2. The blue curve in this plot chronicles growth in Δv_R as *Phobos-Grunt* launch date is delayed from its actual occurrence on 8 November 2011. On 29 November 2011, the Δv_R curve first exceeds the Δv_C limit plotted in gray. Estimated launch season closure is therefore 28 November 2011. For reference, the green curve plots growth in Δv_{TMI}, and the red curve plots growth in Δv_{MOI}.

The estimated 28 November 2011 launch season closure date inferred from Figure 2 data is 8 days later than that previously cited from a RussianSpaceWeb.com report. This deviation may be due to intentionally optimistic assumptions associated with Figure 2 data. However, Roscosmos head Vladimir Popovkin is quoted as stating on 14 November 2011 that *Phobos-Grunt*'s window for Mars departure would close in early December[**]. The 28 November 2011 launch season closure estimate may therefore be considered "in the ballpark", particularly if Δv_{MOI} can be reduced by techniques such as increasing H_A.

But the entire discussion of *Phobos-Grunt* launch season closure is academic, if not intentionally misleading, in the context of actual launch having occurred on 8 November 2011. As will be demonstrated in the next section, that launch imposes a latest mission recovery date far earlier than even 20 November 2011.

Estimated *Phobos-Grunt* Latest Mission Recovery Date

The total change-in-velocity requirement $\Delta v_R'$ associated with *Phobos-Grunt* mission recovery following actual launch on 8 November 2011 is the sum of two components such that $\Delta v_R' = \Delta v_{TMI}' + \Delta v_{MOI}$. For a specified TMI date initiating mission recovery, the Δv_{MOI} component is identical to that required by nominal mission prelaunch planning for that date. But the $\Delta v_{TMI}'$ component will generally require steering through the angle β in order to turn the geocentric *Phobos-Grunt* Earth parking orbit plane into one containing the required Earth departure asymptote bound for Mars. Assuming this steering is done simultaneously with the TMI geocentric speed increase (the most propellant-conservative strategy), associated vector geometry is illustrated in Figure 3.

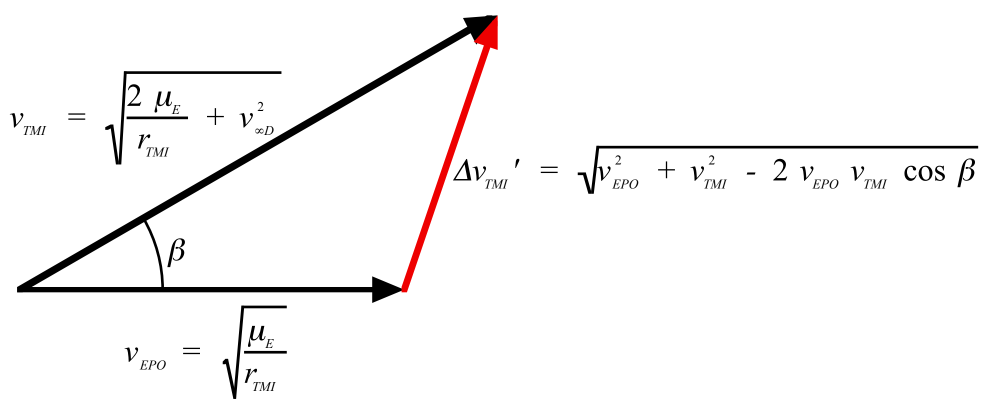

Figure 3. This geocentric velocity vector diagram forms a triangle with sides whose lengths are geocentric speeds. The first side (smaller black arrow) has speed in Earth parking orbit v_{EPO} before TMI. The second side (larger black arrow) has speed in the Earth departure hyperbola v_{TMI} immediately after TMI. The third side (red arrow) has the change-in-velocity magnitude $\Delta v_{TMI}'$ associated with TMI as computed by the law of cosines. When the steering angle β is zero, $\Delta v_{TMI}'$ is simply v_{TMI} minus v_{EPO}, as previously computed for a nominal mission's Δv_{TMI}.

[**] Reference Emily Lakdawalla's blog at http://www.planetary.org/blog/article/00003261/.

Computing β is not a trivial process. Regular USSTRATCOM updates to the as-flown *Phobos-Grunt* trajectory in its Earth parking orbit are processed to determine the spacecraft's angular momentum vector $\boldsymbol{c}$ in geocentric inertial space. Although $\boldsymbol{c}$ is normal to *Phobos-Grunt*'s orbit plane at any instant, excess mass about Earth's equator causes $\boldsymbol{c}$ to precess westward at about 5.4° per day. Meanwhile, asymptotic Earth departure velocity $\boldsymbol{v}_{\infty D}$ is slowly changing with time in geocentric inertial space due to Earth and Mars heliocentric motion. Because it measures the angle between a vector and a plane, β is equivalent to $\boldsymbol{v}_{\infty D}$ latitude with respect to the *Phobos-Grunt* Earth parking orbit plane at a specified mission recovery TMI time. Adopting the sign convention "β is positive when $\boldsymbol{v}_{\infty D}$ points into the hemisphere whose pole is $\boldsymbol{c}$", the following equation computes its value[††].

$$\beta = 90° - \mathrm{acos}\left\{\frac{\boldsymbol{c} \bullet \boldsymbol{v}_{\infty D}}{c\, v_{\infty D}}\right\}$$

The foregoing computational pedigree applies to hypothetical *Phobos-Grunt* mission recovery data summarized in Table 1. From these data, it is evident a *Phobos-Grunt* mission recovery option existed for little more than 3 days after actual launch.

Table 1. Hypothetical *Flagman* propulsion requirement $\Delta v_R'$ for *Phobos-Grunt* mission recovery is tabulated on TMI dates following actual launch on 8 November 2011. Values for Δv_{TMI} are included for comparison purposes with corresponding $\Delta v_{TMI}'$ values because the former assume steering angle β is zero. Because increasing β rapidly inflates $\Delta v_{TMI}'$ as mission recovery TMI is postponed, $\Delta v_R'$ exceeds *Flagman* propulsive capability Δv_C = 4.739 km/s before 12 November 2011.

Parameter	**2011 Date at Hypothetical Mission Recovery TMI**			
	9 Nov	**10 Nov**	**11 Nov**	**12 Nov**
2012 Mars Arrival	11 Sep	11 Sep	12 Sep	12 Sep
β (deg)	+0.260	+3.790	+7.503	+11.204
Δv_{TMI} (km/s)	3.611	3.612	3.613	3.615
$\Delta v_{TMI}'$ (km/s)	3.611	3.665	3.816	4.052
Δv_{MOI} (km/s)	0.858	0.858	0.857	0.857
$\Delta v_R'$ (km/s)	4.469	4.523	4.673	4.909

Inertial dynamics giving rise to β variations can be visualized by projecting snapshots of the precessing *Phobos-Grunt* Earth parking orbit plane onto the geocentric celestial sphere, along with variations in the direction of $\boldsymbol{v}_{\infty D}$. Like the Figure 1 Earth map, north is up and south is down in the Figure 4 celestial sphere plot. In this analogy, Figure 1 latitude is replaced by Figure 4 declination with respect to the Earth mean equator of Julian epoch J2000.0. Likewise, Figure 1 longitude is replaced by right ascension with respect to the mean equinox at J2000.0 in Figure 4. Because Figure 4 shows the inside of a celestial sphere rather than Earth's surface, east is left and west is right. Consequently, the *Phobos-Grunt* orbit plane drifts rightward with time in Figure 4.

[††] A similar quantity called "β" is routinely used in planning International Space Station (ISS) operations, and it has the same sign convention. Of course, this parameter defines $\boldsymbol{c}$ with respect to ISS orbit elements. The only fundamental difference is the ISS β context replaces $\boldsymbol{v}_{\infty D}$ with the Sun's geocentric position vector.

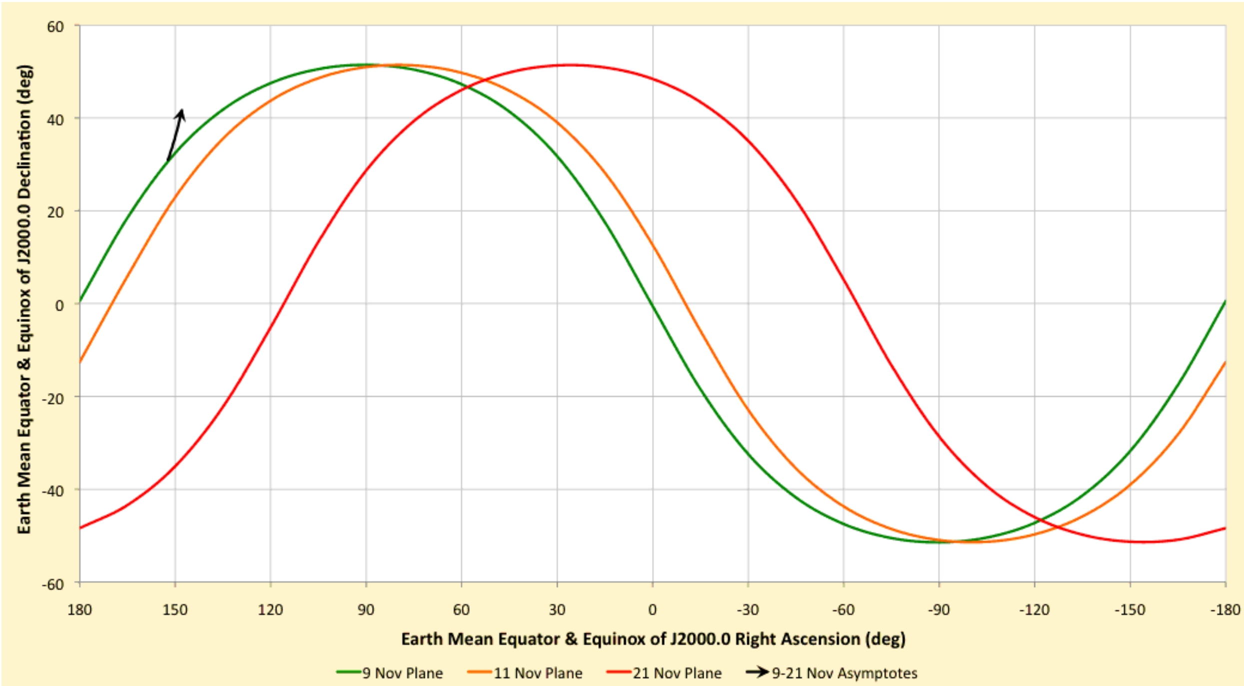

Figure 4. Snapshots of the actual *Phobos-Grunt* Earth parking orbit plane on 9 November 2011 (green), 11 November 2011 (orange), and 21 November 2011 (red) are projected onto the geocentric celestial sphere (truncated at declination magnitudes exceeding 60°) defined by Earth's mean equator and equinox of Julian epoch J2000.0. These snapshots illustrate westward precession of the plane with time. In addition, the slowly changing direction of Earth asymptotic departure for Mars is plotted as a black arrow with the 9 November 2011 direction at the "tail" end and the 21 November 2011 direction at the "head" end. The asymptotic departure locus lies closest to the *Phobos-Grunt* orbit plane on 9 November 2011, shortly after actual launch. Only then are TMI propulsive steering losses due to increasing β negligible. By 11 November 2011, these losses are about to exceed estimated *Flagman* capability to recover the mission.

At the 5.4° per day precession rate, about 5 additional weeks would be required for the 21 November 2011 plane to precess near the plotted asymptotic departure locus in Figure 4. By that time in late December 2011, Earth will have phased too close to Mars opposition for *Phobos-Grunt* mission recovery with *Flagman* propulsive capability. Note also the possibility that asymptotic departure declination can drift so far north (or south) that it exceeds the orbit plane's northern (or southern) declination limit. Under such geometry, TMI β might never be sufficiently near zero, regardless of orbit plane right ascension of the ascending node.

Conclusion

The *Phobos-Grunt* mission's failure to achieve TMI serves as an empirical demonstration of the difference between a launch season and the interval in which a mission may be recovered after an otherwise nominal launch into Earth parking orbit leads to a delayed TMI. An inexorable TMI countdown clock is running during the mission recovery interval and, in the *Phobos-Grunt* case, this clock expired several days after actual launch during a launch season lasting weeks. This situation was never clearly communicated as it played out in November 2011.

But there are broader implications from the *Phobos-Grunt* mission recovery scenario. A similar countdown clock is set following the first of multiple launches required to build up sufficient mass in low Earth orbit (LEO) for departure to any interplanetary destination. An adequately padded launch campaign timeline manages the risk of late departure, but additional exposure to the LEO environment carries its own risks.

Reusable infrastructure in LEO, a propellant depot being a notable example, will be particularly challenged to ensure β is sufficiently near zero at a time when the interplanetary destination is properly phased with Earth. This may require deploying such reusable infrastructure at a sufficiently high orbit inclination to guarantee all conceivable Earth departure asymptote declinations are accommodated. Sufficiently high inclination will generally incur a performance penalty for all launches supporting the reusable infrastructure's logistics. It may therefore be preferable to adopt single-use, mission-specific architectures for multiple-launch interplanetary mission campaigns if they must be staged in LEO.