Orthographic Trajectory Plotting

Consider a trajectory in 3-dimensional space whose positions are expressed as Cartesian coordinates in a "Plot" coordinate system. The system typically has mutually orthogonal axes to which coordinates are referred, but Plot axes may also be cylindrical or even spherical if appropriate limits and "wrapping" logic are applied (as in plotting a trajectory ground track onto a world map, for example). In any case, the trajectory is sampled at discrete positions expressed as $\boldsymbol{r}_P = [x_P, y_P, z_P]^T$ with Plot coordinates. It then remains to transform each Plot position to the orthogonal "Screen" coordinate system for display to the user as $\boldsymbol{r}_S$ coordinates. Screen coordinates are also Cartesian with $x_S > 0$ directed rightward of the display origin, $y_S > 0$ directed upward of the display origin, and $z_S > 0$ directed outward from the display toward the user according to right-handed convention. Because transformed positions are being plotted, as opposed to subtended angles, this process is termed "orthographic".

The user specifies a plot-to-screen transformation M_P^S to provide a desired perspective for trajectory display. In the Figure 1 example, a yaw-pitch-roll (ψ-θ-ϕ or Y-P-R) Euler sequence determining the transformation can be specified graphically via slider controls or digitally via numeric input.

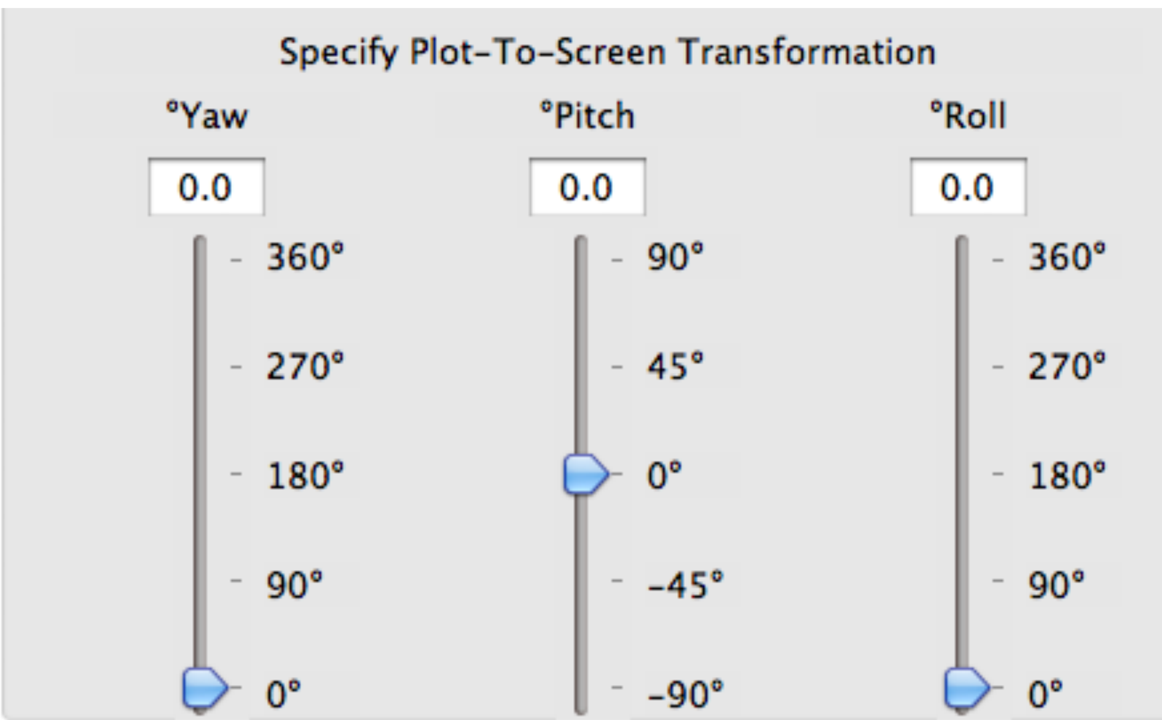

Figure 1. This user interface serves as an example of plot-to-screen transformation specification using graphic and digital input. With these null inputs, an identity transformation is invoked.

Continuing with Figure 1's specification, the ψ-θ-ϕ Euler sequence can be expressed as M_P^S in matrix form.

Orthographic Trajectory Plotting

$$M_P^S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

And the corresponding yaw-pitch-roll product matrix is as follows.

$$M_P^S = \begin{bmatrix} \cos\psi\cos\theta & \sin\psi\cos\theta & -\sin\theta \\ -\sin\psi\cos\phi + \cos\psi\sin\theta\sin\phi & \cos\psi\cos\phi + \sin\psi\sin\theta\sin\phi & \cos\theta\sin\phi \\ \sin\psi\sin\phi + \cos\psi\sin\theta\cos\phi & -\cos\psi\sin\phi + \sin\psi\sin\theta\cos\phi & \cos\theta\cos\phi \end{bmatrix}$$

A trajectory is mapped for display in point-by-point fashion as $\boldsymbol{r}_S = M_P^S\,\boldsymbol{r}_P$. Because a conventional display is 2-dimensional, z_S is effectively zero for all transformed points. But that limitation need not impose complete loss of the trajectory's third dimension through the plotting process. Suppose the projection of a particular $\boldsymbol{r}_P$ onto the Plot coordinate system's x/y plane is to be displayed. After moving the plotting "pen" to $\boldsymbol{r}_S$, a straight line is plotted from that Screen location to Screen coordinates $M_P^S\,[x_P, y_P, 0]^T$ for display of the desired projection. This concept takes its inspiration from hand-drawn diagrams appearing in Guy Ottewell's *The Astronomical Companion* (see https://www.amazon.com/Astronomical-Companion-Guy-Ottewell/dp/0934546606 for examples).

And z_S has application when plotting solid bodies to accompany orthographic trajectories. If terminator, latitude, or longitude points are to be mapped onto a solid body's surface, visible points will satisfy the constraint $z_S > 0$ when the Screen origin coincides with this body's center. Points masked from view by the solid body will have $z_S < 0$. Figure 2 illustrates this logic applied to a day/night terminator, together with parallels of latitude mapped onto a sphere at 20° intervals starting at the equator.

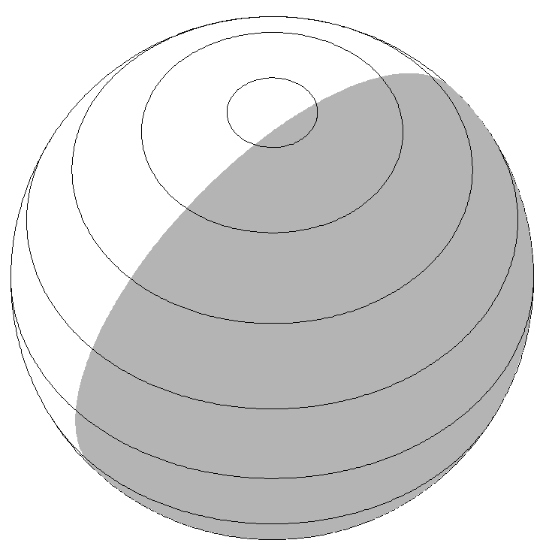

Figure 2. Parallels of latitude are mapped onto a sphere with points hidden when the "negative z_S" criterion is met. Shading indicates the sphere's nightside.

Figure 3 illustrates how *x*/*y* plane projections can become hidden if the Plot projection plane (the ecliptic in this example) coincides with the Screen *x*/*y* plane. In this case, M_P^S is the identity matrix and aligns both planes such that projections onto the ecliptic are seen end-on and become invisible. The arrow annotated "**To** ♈" in Figure 3 corresponds to positive x_P and positive x_S with respect to the Sun origin in both coordinate systems. This arrow also corresponds to the axis of a positive roll Euler rotation ($+\phi$).

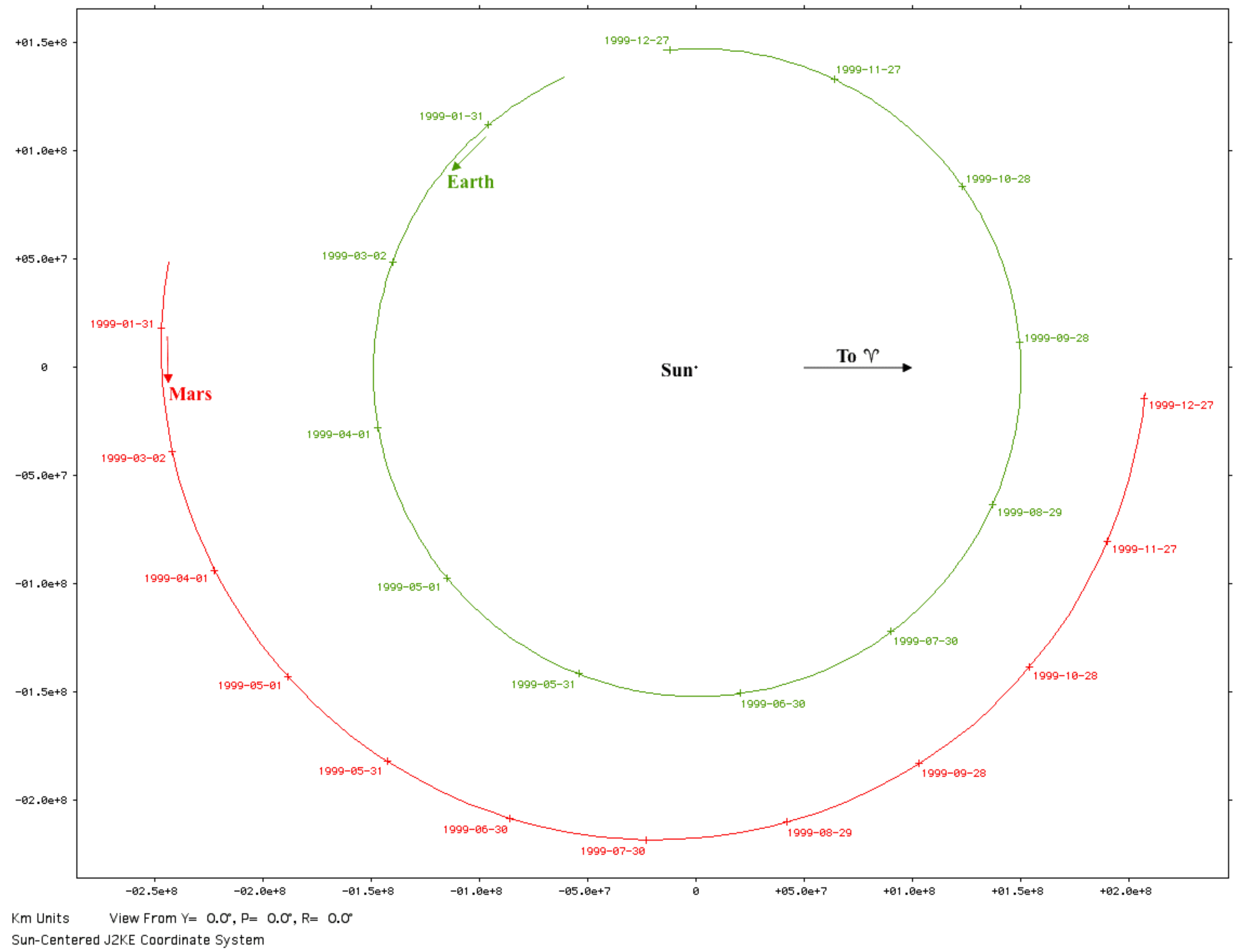

Figure 3. Heliocentric motion of the Earth (green) and Mars (red) is displayed with the *x*/*y* plane coincident to that of Earth's orbit, the ecliptic plane, in both Plot and Screen coordinates. At lower left, note the Plot-to-Screen transformation annotated as a null yaw-pitch-roll (Y-P-R) Euler sequence.

In Figure 3, perspective is from the ecliptic pole at which $+z_P$ is directed, and ecliptic declination $\delta = +90°$. Define ecliptic longitude λ as a heliocentric angle whose value increases in the counterclockwise sense from zero in the ♈ direction. Now imagine rotating Figure 3 perspective through a roll $\phi = +45°$ about the heliocentric Plot origin such that $+z_S$ points in a direction equivalent to $\lambda = 270°$ and $\delta = +45°$. Figure 4 illustrates the result of this changed perspective. Projections onto the ecliptic plane remain invisible for Figure 4 Earth heliocentric motion because it cannot significantly depart from the ecliptic at this scale. But the heliocentric orbit plane of Mars is inclined more than 1.8° to the ecliptic. Stubby dotted projection lines are visible in Figure 4 from date-annotated "+" Mars position markers with sufficient displacement from the ecliptic. In this manner, Figure 4 projection lines give a meaningful impression of how the heliocentric orbit plane of Mars is oriented in 3-dimensional space with respect to the ecliptic plane.

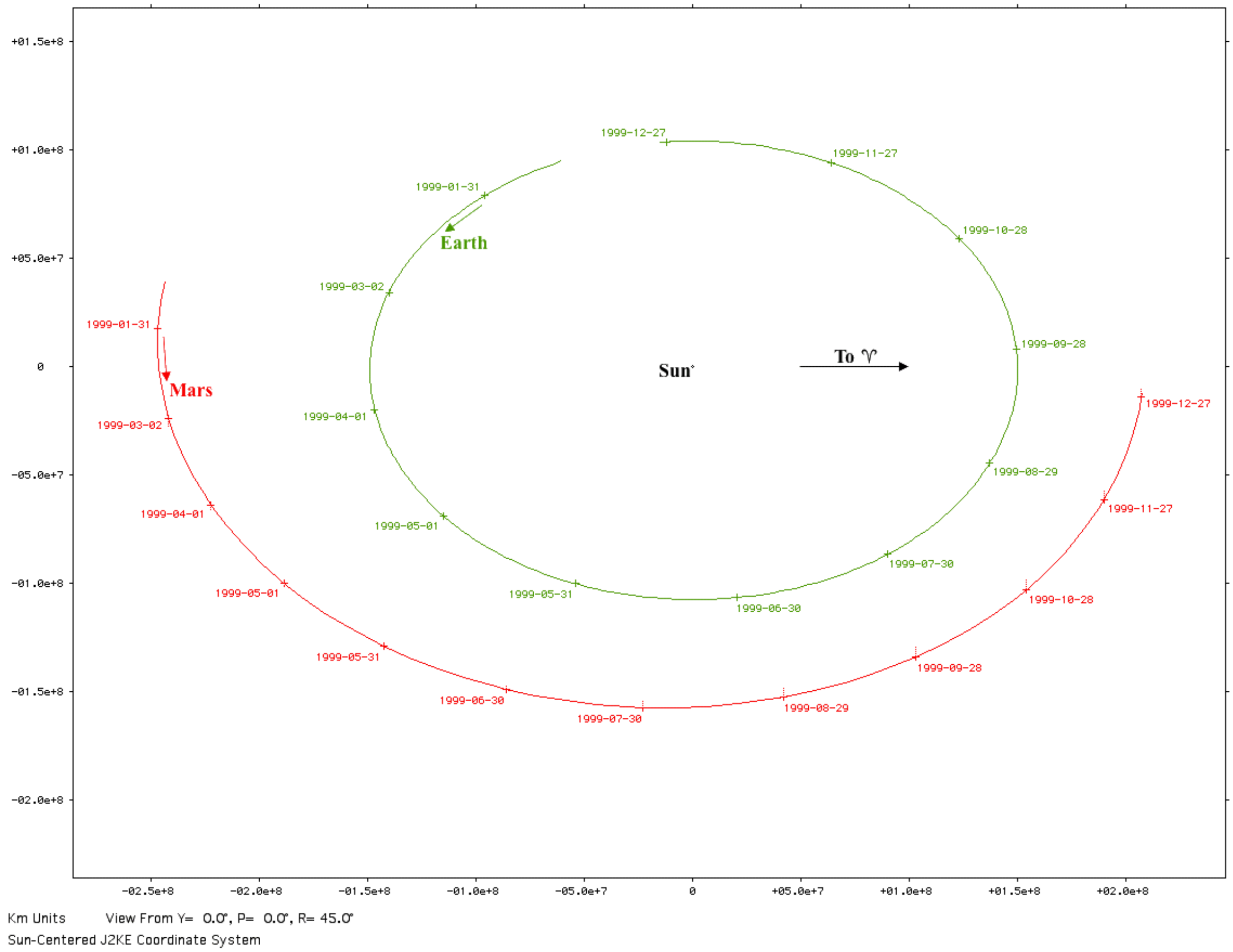

Figure 4. With respect to Figure 3's perspective, this view of Earth (green) and Mars (red) heliocentric motion has been rotated +45° about the axis annotated "To ♈". This perspective foreshortens both planets' orbits in the vertical direction, and it also enables perception of Mars motion with respect to the ecliptic plane near which Earth orbits the Sun. Dotted projection lines onto the ecliptic extend from each date-annotated "+" Mars position marker. These projections indicate Mars has a descending node on the ecliptic plane circa 1999-05-31. No projection lines are visible for Earth because its orbit has no deviation from the ecliptic plane at this scale.