GRAIL Takes A Roundabout Route To Lunar Orbit

The Gravity Recovery and Interior Laboratory (GRAIL) mission departed Earth from Cape Canaveral Air Force Station Space Launch Complex 17B on 2011 September 10 at 13:08 UTC, carried by a Delta II Heavy launch vehicle. Liftoff occurred on the third day of a launch season lasting 42 consecutive days.

Twin spacecraft, currently known as GRAIL-A and GRAIL-B, were launched aboard the Delta II Heavy. Each had a mass of 307 kg at launch, 106 kg of which was helium and hydrazine for trajectory changes by a single 22-N main engine and for attitude control by eight 0.9-N thrusters. Beginning 2012 March 8, the spacecraft will conduct their 82-day science mission (spanning 3 lunar sidereal rotations) while flying in the same polar lunar orbit, initially at a circular height of 55 km, with GRAIL-B leading GRAIL-A. Mean separation between the spacecraft will be controlled between 65 km and 225 km, in rough proportion to orbit height. Because the spacecraft are solar-powered, their science mission is constrained to fall between the total lunar eclipse of 2011 December 10 and the partial lunar eclipse of 2012 June 4.

As GRAIL's name implies, the mission's primary objective is to map the Moon's gravity field, improving nearside knowledge by a factor of 100. Because no spacecraft has ever been tracked when hidden from Earth over the Moon's farside, GRAIL is expected to improve knowledge there by a factor of 1000. Neither GRAIL-A nor GRAIL-B will be tracked from Earth while orbiting over most of the Moon's farside, but the two spacecraft will circumvent this problem by tracking each other over radio links between them. This interface will conduct Ka-band ranging, while timing data are exchanged via S-band. An onboard gravity recovery processor assembly produces radiometric data for downlink when the Moon is not blocking transmissions to Earth.

Perhaps even more interesting to astrodynamicists are the routes GRAIL-A and GRAIL-B will take to reach the Moon. These trajectories are the focus of this article. Although their destination is but 400,000 km from Earth, each GRAIL spacecraft will travel about 10 times that distance with respect to Earth before reaching initial lunar orbit. The circuitous routes GRAIL-A and GRAIL-B will take to the Moon require 112.4 days and 113.4 days, respectively. Approaching the Moon over its south pole, each will perform a 38-minute lunar orbit insertion (LOI) burn, GRAIL-A's at 2011 December 31.9 UTC and GRAIL-B's about 25 hours later.

It would have been possible for the Delta II Heavy to deliver both spacecraft to the Moon in 3 or 4 days, but a more leisurely route is desirable for multiple reasons. First, the uninterrupted season of 42 viable launch days would not have been possible with short lunar transits from Earth. Second, extended trans-lunar cruise permits spacecraft systems to be thoroughly checked and calibrated well in advance of LOI. Third, LOI propulsion requirements are appreciably reduced using a weak stability boundary interaction obtained by flying to the edge of interplanetary space near the first Sun-Earth libration point (SEL1).

Located 1.5 million km from Earth in the sunward direction, SEL1 is a quasi-stable "saddle" formed as Earth's relatively minute gravity field gives way to the Sun's. Unstable motion, equivalent to moving on the saddle's convex contour from stirrup to stirrup, is along the Earth-Sun line. Stable motion occurs transverse to this line, equivalent to the saddle's fore and aft concave contour. Motion near SEL1 is therefore chaotic in nature: a small displacement in position or increment in velocity can exert disproportionally large trajectory changes after a sufficient time. Since both GRAIL spacecraft are able to apply small trajectory changes independently from each other whenever necessary, SEL1's instability can be turned into an advantage as illustrated by Figure 1.

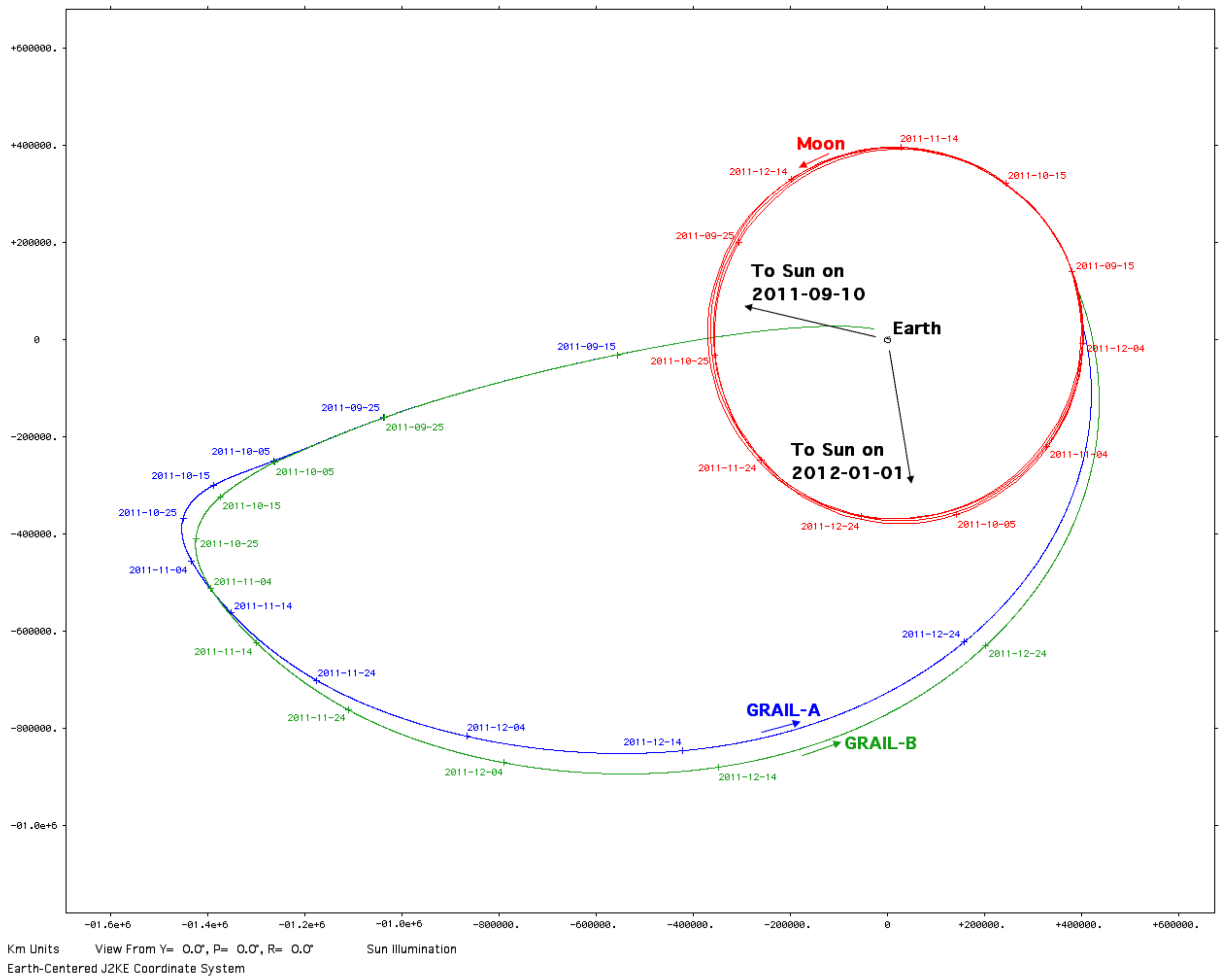

Figure 1. Geocentric GRAIL trajectories plotted in the inertial ecliptic plane. Time tick labels are in year-month-day format.

Although both spacecraft leave Earth's vicinity in close proximity at Figure 1's scale, they naturally separate from each other as SEL1 is approached in October of 2011. Because Figure 1 is plotted in an inertial geocentric coordinate system, the sunward direction from Earth rotates counterclockwise by more than 90° during GRAIL's trans-lunar cruise. Two black arrows pointing away from Earth illustrate this rotation in Figure 1.

The Moon makes over 4 complete counterclockwise revolutions about Earth in Figure 1 during trans-lunar cruise. Note that, when the two spacecraft reach their destination, they are travelling in very nearly the same geocentric direction as the Moon. This condition helps minimize LOI propulsion requirements. Furthermore, both lunar rendezvous points fall very nearly 90° counterclockwise in the Moon's orbit with respect to the sunward direction from Earth at that time. This condition equates to the Moon being near first quarter phase as seen from Earth. A lunar rendezvous near first quarter phase is the most direct approach from SEL1 resulting in motion with the Moon in its orbit. According to the U.S. Naval Observatory, the Moon reaches first quarter phase at 2012 January 1.26 UTC, between the two planned LOI burns.

Figure 2 illustrates details of GRAIL trajectories near SEL1. Because this plot is with respect to a geocentric coordinate system rotating at the rate Earth revolves about the Sun, SEL1 corresponds to a fixed point as annotated in Figure 2.

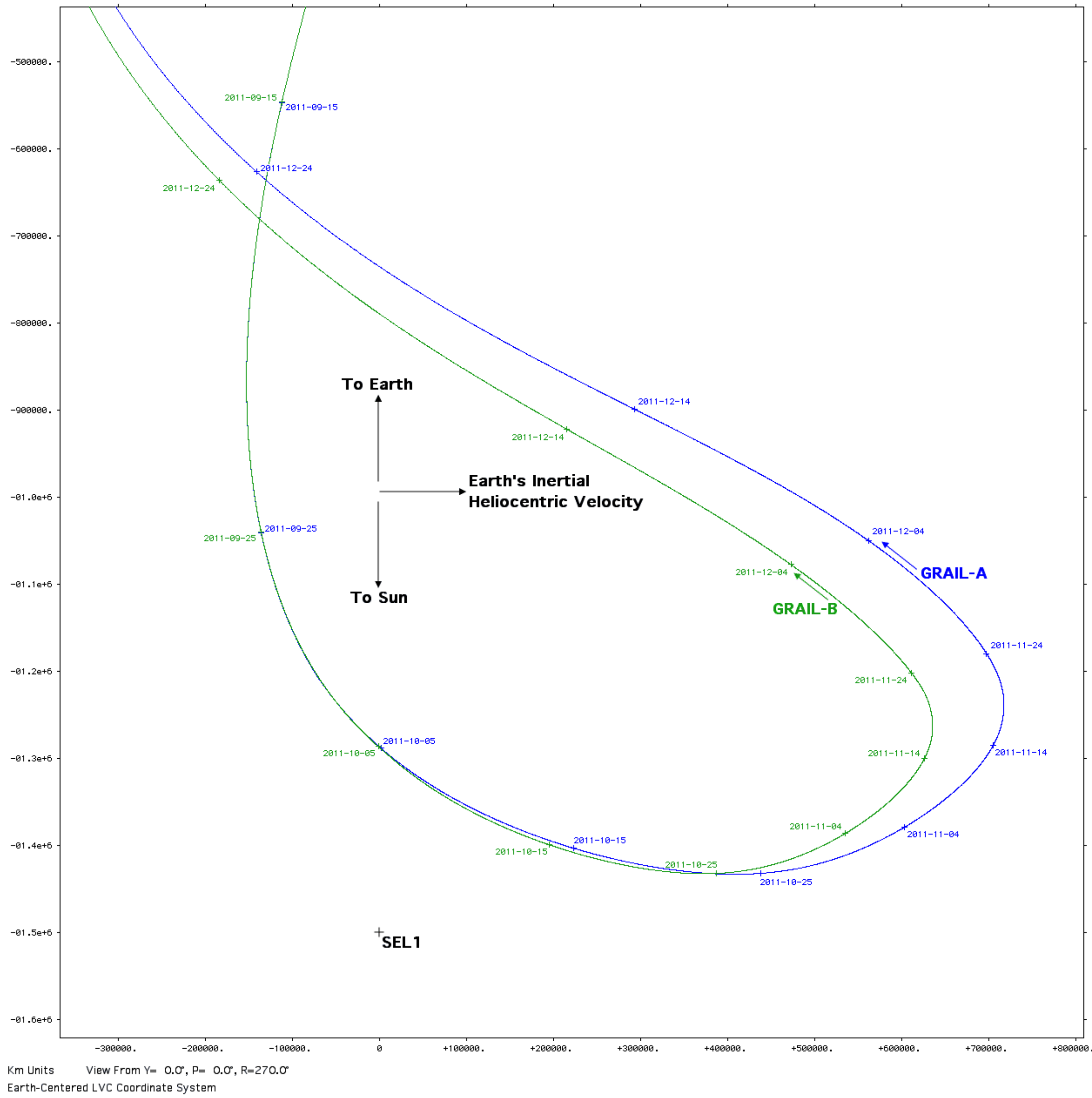

Figure 2. Geocentric GRAIL trajectories in the ecliptic plane plotted near SEL1 using a coordinate system rotating as Earth revolves about the Sun. Time tick labels are in year-month-day format.

Returning to the SEL1 saddle analogy in Figure 2's context, the reversal in GRAIL spacecraft motion initially away from Earth is achieved by not travelling in the sunward direction from Earth farther than SEL1. Cresting the saddle beyond SEL1 along an unstable radial contour would almost certainly result in departing Earth for interplanetary space. Depending on the rate of departure, return to Earth's vicinity could occur in a few months or perhaps not for centuries, if ever. Chaos truly reigns near quasi-stable libration points, particularly when motion is slow.

Another means of gaining insight regarding GRAIL trans-lunar coast is presented in Figure 3, which plots geocentric osculating semi-major axis for each spacecraft as a function of time.

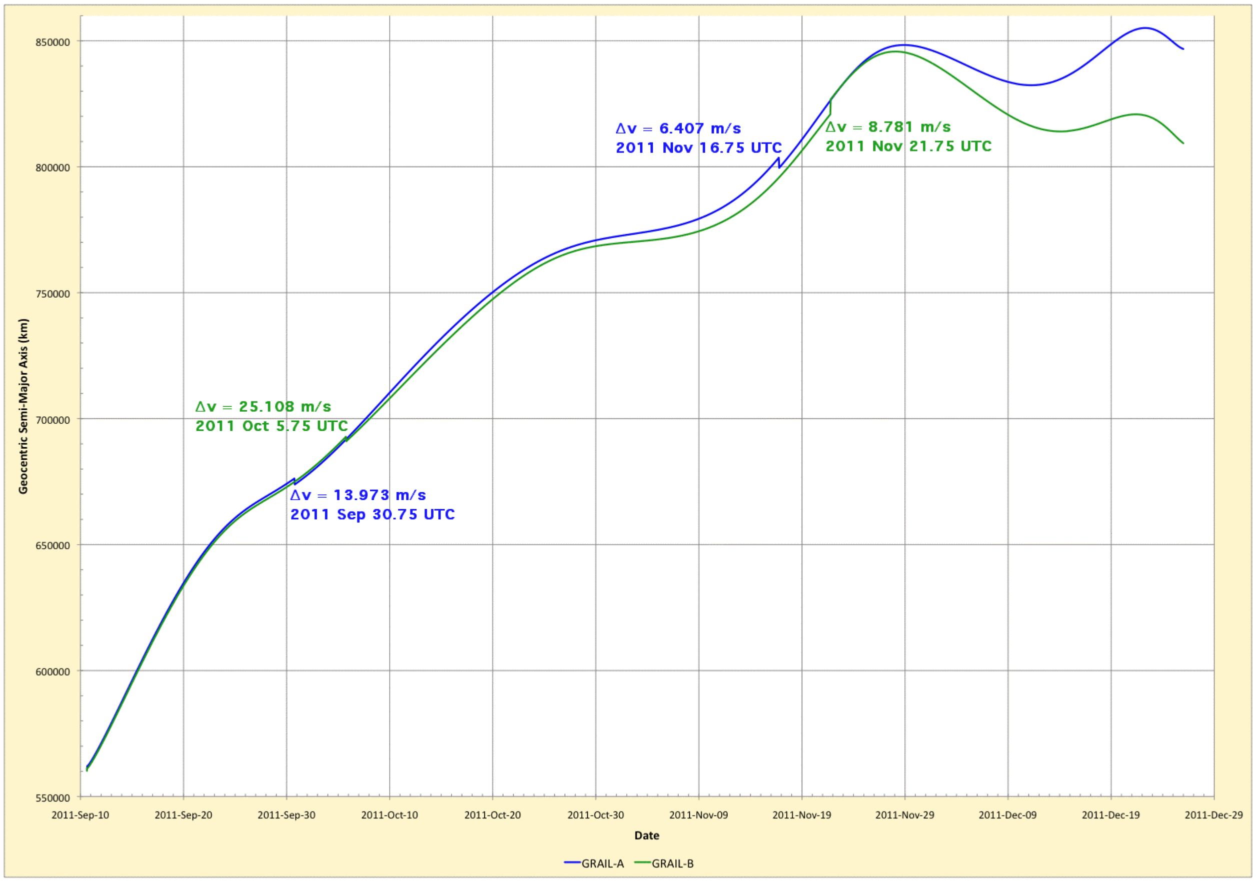

Figure 3. Geocentric osculating semi-major axis versus time plotted for each GRAIL spacecraft during trans-lunar coast.

An unperturbed geocentric conic trajectory would contribute a horizontal line to Figure 3, but solar and lunar gravity perturbations play major roles in GRAIL trajectory dynamics with respect to Earth. Relatively small discontinuous jumps in semi-major axis are attributable to planned spacecraft course corrections. These are annotated with an inferred change in velocity (Δv) and UTC from GRAIL trajectory targeting posted for public access on 2011 September 23.

Terminal lunar approaches by both GRAIL spacecraft are illustrated in Figure 4, together with initial orbits following LOI. Each LOI burn arc is approximately centered on pericynthion, which falls near ascending node on the lunar equator. The shaded portion of the Moon in Figure 4 is its farside, so the plot is viewed in a direction very nearly toward Earth. Since the Moon is in first quarter phase, the Sun's illumination would be from the left, and the Moon's sunset terminator would run very nearly vertically to bisect the Moon's disc in Figure 4.

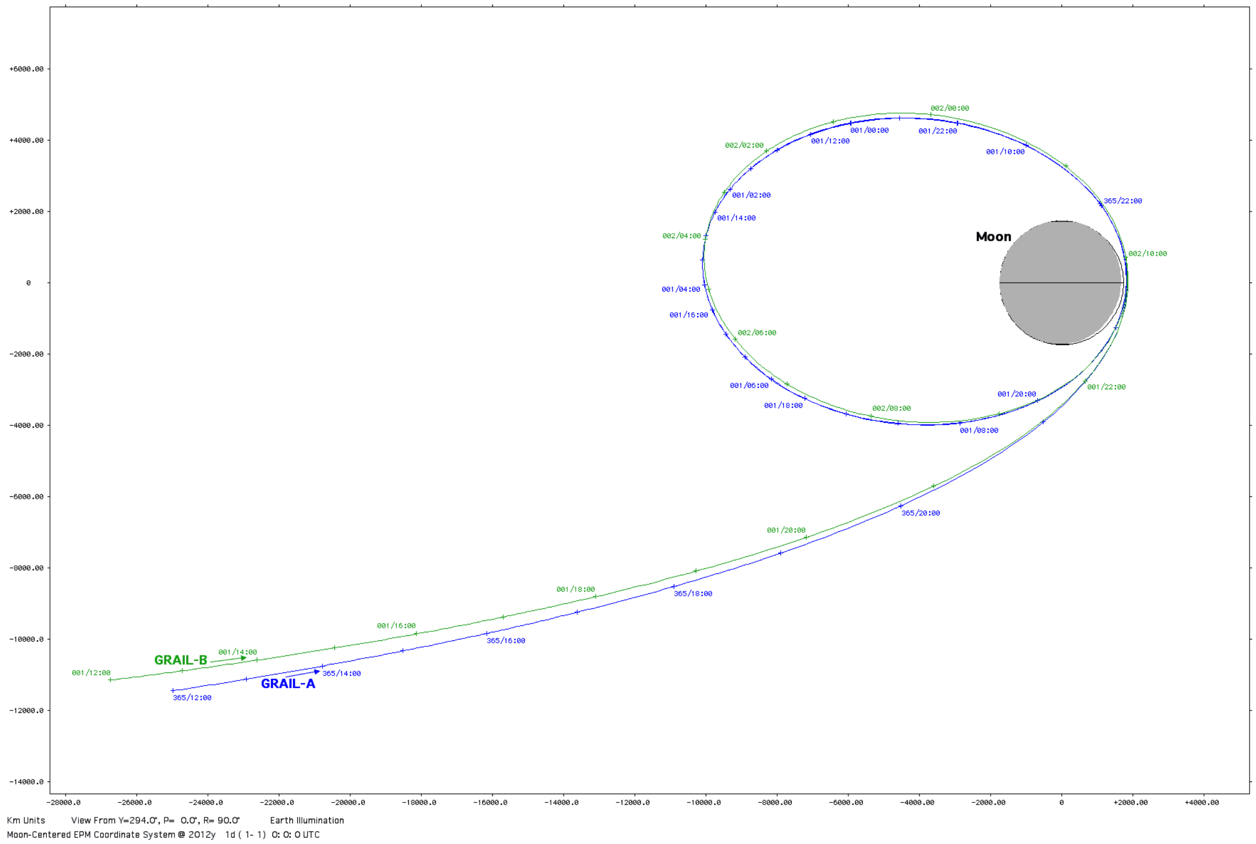

Figure 4. Selenocentric GRAIL lunar terminal approaches and initial orbits with the plot plane near both trajectory planes. The Moon's farside with respect to Earth is shaded. Time tick labels are UTC in 2011 and 2012 day-of-year/hour:minute format.

Coasted pericynthion heights neglecting LOI are 137 km for GRAIL-A and 156 km for GRAIL-B. Most Apollo lunar approach pericynthion heights were near 100 km, but transit time from Earth was never far from 3 days. This invites comparisons between GRAIL's lunar approach speeds and analogous as-flown reconstructions from Apollo. Such comparisons are facilitated by the conic *vis viva* energy integral as provided in Equation 1.

$\mu \equiv$ Moon's reduced mass = 4902.798 km^3/s^2

$r \equiv$ lunar speed comparison arbitrary selenocentric distance = 1837.53 km (equivalent to a lunar height of 100 km)

$a \equiv$ case-specific selenocentric semi-major axis

$$s \equiv \text{case-specific selenocentric speed at } r = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)} \quad (1)$$

Three as-flown Apollo cases are compared with the planned GRAIL lunar approaches in Table 1. Because Apollo 13 suffered a trans-lunar abort and never performed LOI, its case-specific *a* is obtained prior to a "pericynthion +2 hours" (PC+2) burn.

Table 1. Selenocentric speed (*s*) comparisons referencing as-flown Apollo and planned GRAIL lunar approach trajectory cases at an arbitrary 100 km height.

Case	*a* (km)	*s* (km/s)
Apollo 10 Pre-LOI	-4180.657	2.551279
Apollo 13 Pre-PC+2	-4473.879	2.536171
Apollo 15 Pre-LOI	-4580.352	2.531144
GRAIL-A Pre-LOI	+152,177.134	2.303058
GRAIL-B Pre-LOI	+184,112.045	2.304270

A dramatic difference between Apollo and GRAIL cases is evident from their Table 1 a values. With Apollo cases, the $a < 0$ condition indicates a lunar flyby leading to escape will occur if a braking impulse is not applied. Assuming conic selenocentric motion, the $a > 0$ condition applicable to GRAIL cases indicates capture into an elliptical orbit has occurred before any LOI impulse. But the large magnitude of GRAIL a values, greatly exceeding the 67,000 km radius of the Moon's gravitational sphere of influence, is testimony to their common SEL1 pedigree. Without at least a partial LOI, each GRAIL spacecraft will escape the Moon after a close flyby.

Table 1 s values confirm the roundabout GRAIL route to the Moon will reduce LOI Δv by about 240 m/s with respect to faster transits from Earth suggested by Apollo as-flown trajectories. Considering it would require Δv near 670 m/s for each GRAIL spacecraft to achieve a circular lunar orbit at 100 km height from their respective approach trajectories, this savings is significant.

All planned GRAIL trajectory data in this article are obtained from JPL's *Horizons* ephemeris computation service at http://ssd.jpl.nasa.gov/?horizons. Background information on the GRAIL mission is documented in a launch press kit available for download at http://www.jpl.nasa.gov/news/press_kits/graiLaunch.pdf. The GRAIL mission home page can be accessed at http://solarsystem.nasa.gov/grail/home.cfm, and additional information is available at http://moon.mit.edu/. Lunar phase predictions may be obtained from http://aa.usno.navy.mil/data/docs/MoonPhase.php. Foregoing URLs were each accessed on 2011 September 23.