Reproducing an Apollo Applications Program Single-Launch Human Venus Flyby Trajectory

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As proposed to the Apollo Applications Program by NASA's Manned Spacecraft Center in 1967, a single Saturn 5 launch of Apollo-derived hardware could send a crew of three on a Venus flyby mission with free return to Earth one year later. Three 30-day Earth departure seasons between 1972 and 1975 were identified. The season-open trajectory for Earth departure on 4 April 1972 is developed in detail by this paper with the objective of confirming salient features of the 1967 proposal using modern trajectory design and visualization techniques.

I. Nomenclature

H	=	height above a planet's equatorial radius
J_{20}	=	Legendre polynomial coefficient of degree 2 and order zero relating to Earth's excess equatorial mass
d_R	=	Earth return date
d_V	=	Venus flyby date
h	=	geodetic altitude
$\boldsymbol{r}$	=	position vector
$\boldsymbol{v}$	=	velocity vector
v_{EI}	=	geocentric inertial speed at Earth atmospheric entry interface (defined at H = +121.92 km)
$\boldsymbol{v}_{\infty A}$	=	planet-relative asymptotic arrival velocity from a heliocentric trajectory
$v_{\infty A}$	=	planet-relative asymptotic arrival speed from a heliocentric trajectory
$\boldsymbol{v}_{\infty D}$	=	planet-relative asymptotic departure velocity to a heliocentric trajectory
$v_{\infty D}$	=	planet-relative asymptotic departure speed to a heliocentric trajectory
Δv	=	change-in-velocity magnitude
δ	=	declination with respect to a planet's true equator
γ_{EI}.	=	inertial flight path angle at Earth atmospheric entry interface (defined at h = +121.92 km)
λ	=	true equatorial longitude with respect to a planet's prime meridian
ρ_A	=	areal density
ψ	=	velocity heading in the local horizontal plane measured with respect to true north
θ	=	interplanetary trajectory heliocentric transfer angle from departure terminus to arrival terminus

II. Introduction

In February 1967, year-long human Venus flyby missions were proposed by the NASA Manned Spacecraft Center† (MSC) as an option to be considered by the Apollo Applications Program (AAP)‡. Details of the proposal are documented in an MSC internal note [1]. Remarkably, these flybys could be initiated with but a single launch using the same 3-stage Saturn 5 launch vehicle then designed to enable Apollo Program lunar landings. The flyby mission spacecraft differs from that for an Apollo lunar landing in only two major respects [1, p. 1]. First, the Lunar

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† This facility near Houston, TX was renamed the Lyndon B. Johnson Space Center (JSC) in 1973.
‡ Reference the article titled "2012 Venus Transit Special #1: Piloted Single-Launch Venus Flyby (1967)" by David S. F. Portree at http://www.wired.com/2012/05/2012-venus-transit-special-1-piloted-single-launch-venus-flyby-1967/ (accessed 9 January 2016).

Module is replaced with a Mission Module (MM) serving as the 3-person crew's primary habitat throughout interplanetary flight. Second, the Command Module's (CM's) heat shield is upgraded to withstand loads from Earth atmospheric entry at speeds up to 13.7 km/s[§]. This upgrade is with respect to Apollo Program entry speeds from cislunar space at approximately 11 km/s [2, p. 581].

The proposed single-launch mission mode critically depends on assuming "the MM is the Apollo Orbiting Laboratory" [1, p. 1]. According to this proposal [1, p. 9, Table II], the MM has a total mass of 12,579 kg with 991 kg providing an areal density ρ_A = 0.98 g/cm^2 for meteoroid protection. An effective MM area of 101.45 m^2 is inferred from the meteoroid shielding values. When this area is divided into MM total mass, ρ_A = 12.40 g/cm^2 is obtained as a crew radiation shielding metric. Likely because the Apollo Orbiting Laboratory was intended for operation exclusively in low Earth orbit (LEO), ρ_A = 12.40 g/cm^2 compares well with other LEO habitats. According to NASA[**], the International Space Station's cylindrical *Destiny* module has a length of 8.53 m, a diameter of 4.27 m, and a mass of 14,500 kg. Consequently, a modern LEO habitat like *Destiny*, designed for astronaut sojourns of up to a year, has an effective ρ_A = 10.15 g/cm^2. The AAP's *Skylab* Orbital Workshop, serving as LEO habitat for crew visits up to 3 months' duration in 1973 and 1974, had an effective ρ_A = 9.34 g/cm^2 [3, p. 62][††].

To comply with current astronaut radiation exposure standards, a crew habitat with ρ_A = 51.5 g/cm^2 (equivalent to 5% ρ_A from Earth's atmosphere at sea level) has been proposed for the interplanetary transport *Aquarius* whose roundtrips to Mars would spend about 16 months between planets [4, p. 4]. A 4-fold increase in MM mass to match *Aquarius* ρ_A would likely require a second Saturn 5 launch, Apollo Service Module modifications to accommodate additional propellant, and assembly of the MM (presumably the first launch's payload) and modified Command-Service Module (CSM, presumably the second launch's payload) in orbit prior to Earth departure. Under modern astronautic standards, MM ρ_A = 12.40 g/cm^2 would be a difficult interplanetary habitat design to justify based on the perceived value of a human Venus flyby. This ethical dilemma may have been a primary reason the MSC-proposed mission was not adopted for implementation by the AAP. There is also no flight test evidence demonstrating a crew debilitated by a year in microgravity could survive cardiovascular stresses from Earth atmospheric entry at more than 13 km/s, even if the contemplated CM heat shield upgrade could be produced. Nevertheless, reproducing the proposed mission's trajectory with modern mission design and visualization techniques has historic merit, as this paper will demonstrate.

III. Heliocentric Conic Trajectory Reproduction

Three Venus flyby mission timelines with Earth departures on 4 April 1972, 14 November 1973, and 7 June 1975 are documented in the MSC proposal [1, p. 11, Table IV]. The first of these is selected for reproduction, and its as-proposed timeline appears in Table 1. Evident from this timeline is an Earth-to-Venus (Leg 1) trajectory with a "short way" heliocentric transfer angle $0 < \theta < 180°$. To compensate for mean heliocentric angular motion faster than Earth's during Leg 1, the Venus-to-Earth (Leg 2) trajectory must be a "long way" transfer with $180° < \theta < 360°$.

[§] Numeric values recorded in English units by external references are converted to metric units when reported herein.

[**] Reference http://www.nasa.gov/mission_pages/station/structure/elements/destiny.html (accessed 10 January 2016).

[††] The *Skylab* Orbital Workshop has length 14.63 m, diameter 6.71 m, and mass 35,380 kg. This mass likely includes a contribution from *Skylab*'s primary solar arrays (one of which was lost during powered ascent to orbit), but these arrays contributed little to crew radiation shielding. Nevertheless, ρ_A = 9.34 g/cm^2 reflects their presence.

Table 1. Key events in the Venus flyby mission timeline to be reproduced are transcribed from the MSC proposal. Phase elapsed time (PET) is the interval since Earth departure's trans-Venus injection (TVI) impulse.

Date	PET (days)	Event
04/04/1972	0	TVI change-in-velocity magnitude Δv = 3.682 km/s[‡‡]
07/22/1972	109	Venus flyby
03/29/1973	359	Earth return atmospheric entry interface at inertial speed v_{EI} = 13.716 km/s

The reproduction adopts a 4 April 1972 TVI date as a fixed constraint. It then seeks Venus flyby and Earth return dates d_V and d_R, respectively, such that the heliocentric conic Leg 1 trajectory has a Venus-relative arrival asymptotic speed $v_{\infty A}$ very nearly matching Venus-relative departure asymptotic speed $v_{\infty D}$ on the heliocentric conic Leg 2 trajectory. This $v_{\infty A} \cong v_{\infty D}$ condition indicates existence of a Venus flyby whose gravity accelerations can provide a coasted return to Earth with only minor midcourse correction impulses required to address unpredicted trajectory perturbations. Table 2 lists $v_{\infty A} \cong v_{\infty D}$ matches at a granularity of one day for d_V near 22 July 1972, as suggested by Table 1.

For each Leg 1/2 trajectory pair associated with Table 2, Venus-relative arrival/departure asymptotic velocities are computed and used to infer a Venus-centered hyperbolic conic flyby trajectory through the patched conic approximation [5, pp. 359-379]. Each approximation is in turn associated with a Venus flyby periapsis height, and these values appear in Table 2's ***H* Flyby** column.

Table 2. Assuming Earth departure on 4 April 1972, Earth return dates d_R are listed for which Venus arrival/departure asymptotic speed matches $v_{\infty A} \cong v_{\infty D}$ exist on Venus flyby dates d_V near 22 July 1972. Satisfying $v_{\infty A} \cong v_{\infty D}$ is a necessary condition for coasted return to Earth with only small midcourse correction propulsive maneuvers required following TVI.

d_V	d_R	$v_{\infty A}$ (km/s)	$v_{\infty D}$ (km/s)	*H* Flyby (km)	v_{EI} (km/s)
07/17/1972	03/12/1973	6.116	6.117	-2967.7	13.616
07/18/1972	03/16/1973	5.966	5.930	-2486.4	13.627
07/19/1972	03/19/1973	5.823	5.831	-2110.5	13.647
07/20/1972	03/23/1973	5.687	5.660	-1471.1	13.664
07/21/1972	03/26/1973	5.559	5.569	-967.8	13.683
07/22/1972	03/30/1973	5.439	5.413	-119.0	13.697
07/23/1972	04/02/1973	5.327	5.330	+550.3	13.713
07/24/1972	04/05/1973	5.223	5.250	+1299.9	13.726
07/25/1972	04/09/1973	5.128	5.118	+2539.6	13.737
07/26/1972	04/12/1973	5.044	5.048	+3507.9	13.755
07/27/1972	04/15/1973	4.969	4.982	+4572.3	13.774

Noting Table 1's d_V = 22 July 1972, Table 2 indicates this is the latest Venus flyby date and slowest flyby speed at which v_{EI} < 13.7 km/s is in accord with CM upgraded heat shield design. Unfortunately, 22 July is also the last Table 2 d_V at which *H* Flyby is negative. A 23 July 1972 Venus flyby date is therefore adopted for trajectory reproduction.

To target Venus arrival on 23 July 1972, the $v_{\infty D}$ associated with Leg 1's Earth departure on 4 April 1972 is 3.605 km/s. Assuming this departure is from a circular Earth parking orbit with *H* = +185 km [1, p. 8, Table I], total energy invariance in conic trajectories [5, p. 15] imposes a TVI estimated Δv = 3.803 km/s. This value is 0.121 km/s or 3.3% greater than that transcribed in Table 1 from the MSC proposal.

Heliocentric motion of Earth, Venus, and the CSM+MM spacecraft is plotted for Leg 1 in Figure 1 and Leg 2 in Figure 2. Salient features of these plots agree well with a similar mission's heliocentric trajectory plot appearing in the MSC proposal [1, p. 17, Figure 5].

[‡‡] Note this value is almost certainly associated with TVI performed starting in a nearly circular LEO and is inconsistent with the proposed plan to perform TVI from an elliptical Earth orbit with 2-day period [1, p. 3]. Departure from an elliptical Earth orbit is incorporated by the trajectory reproduction in Section IV.

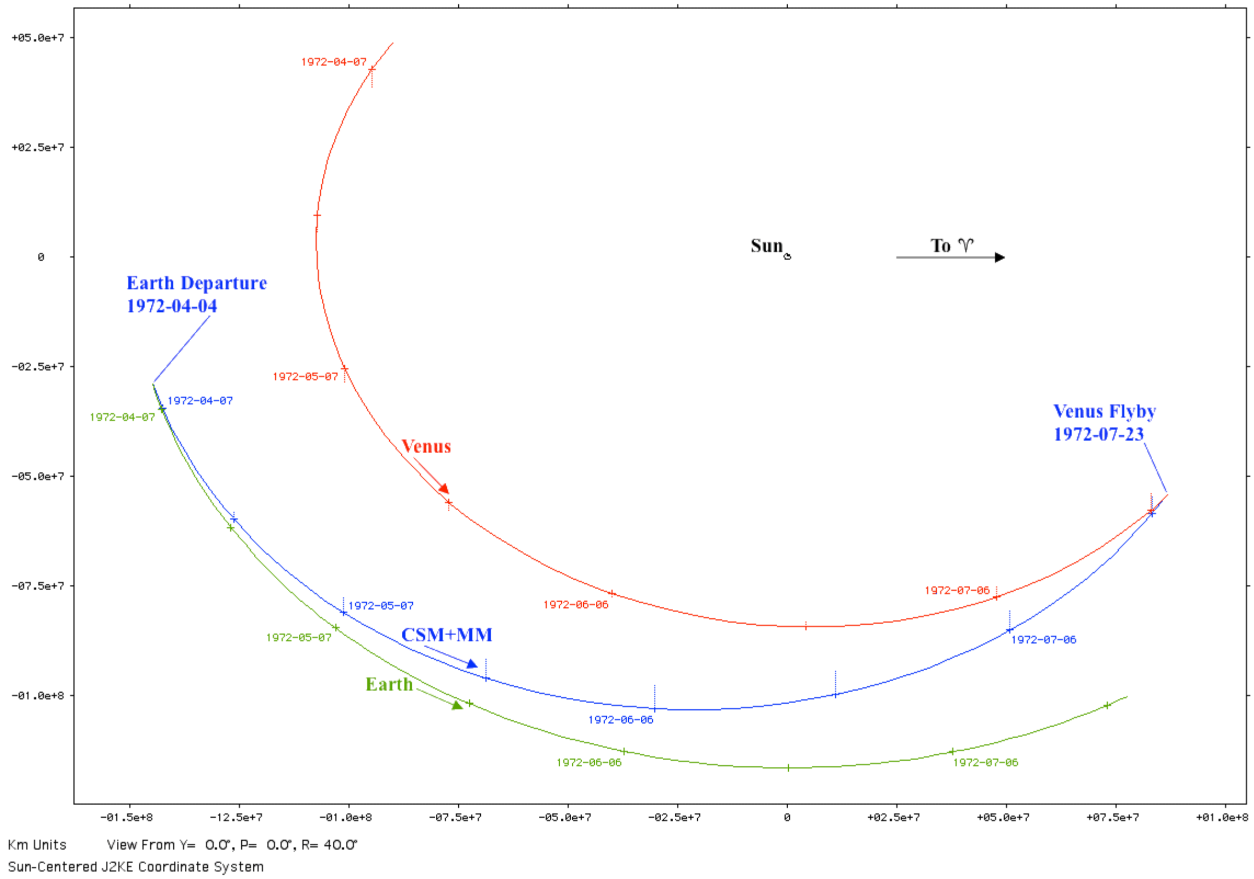

Figure 1. Heliocentric motion of the Earth (green), Venus (red), and CSM+MM spacecraft (blue) is plotted for Leg 1's Earth-to-Venus trajectory reproduction. The view is 40° from normal to the ecliptic plane. Date annotations are in YYYY-MM-DD format. Time ticks ("+" markers) are at 15-day intervals, and dotted lines are projections from these markers onto the ecliptic plane.

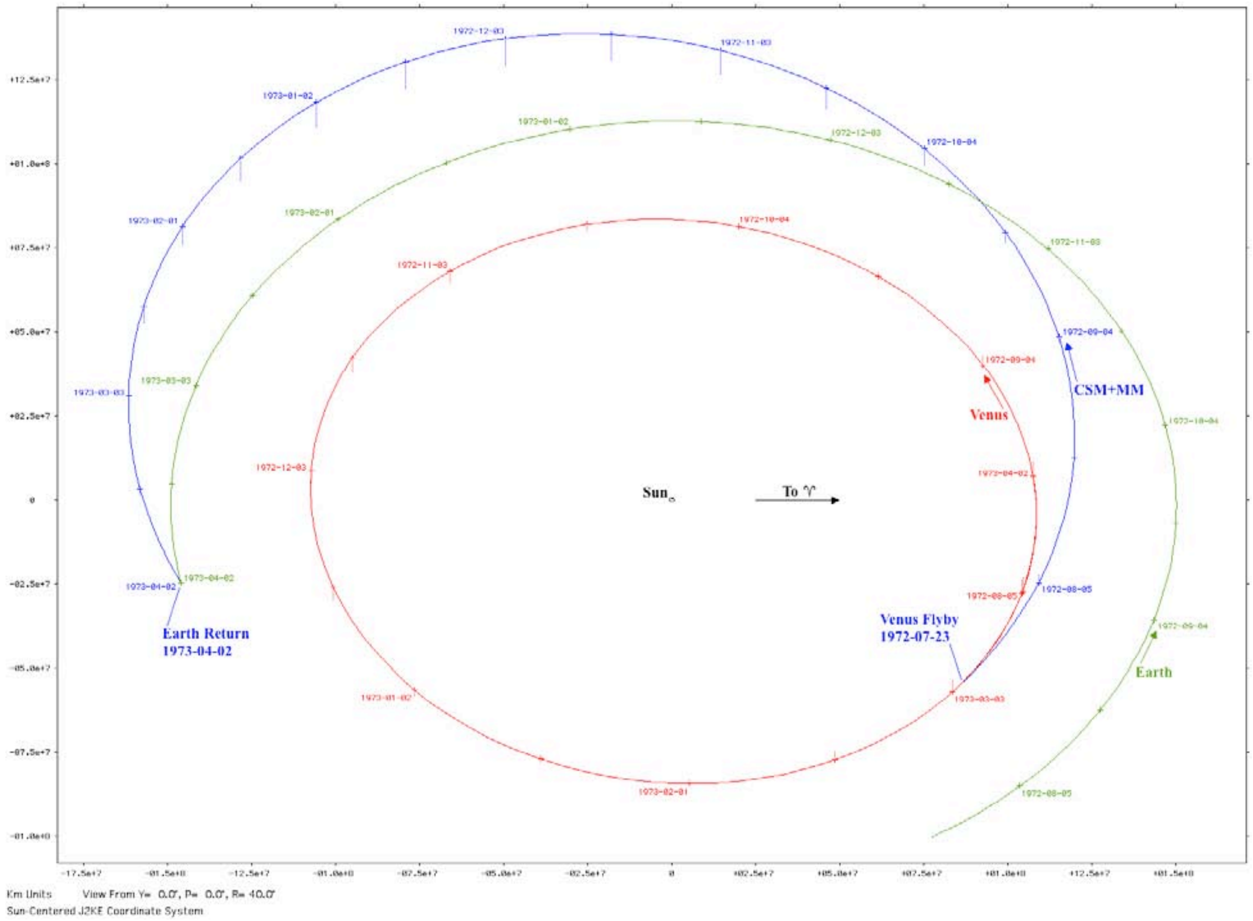

Figure 2. Heliocentric motion of the Earth (green), Venus (red), and CSM+MM spacecraft (blue) is plotted for Leg 2's Venus-to-Earth trajectory reproduction. The view is 40° from normal to the ecliptic plane. Date annotations are in YYYY-MM-DD format. Time ticks ("+" markers) are at 15-day intervals, and dotted lines are projections from these markers onto the ecliptic plane.

IV. Precision Trajectory Reproduction

The conic trajectory reconstruction documented in Section III serves to seed precision trajectory refinements detailed in the next three subsections. Precision trajectory generation is performed using the WeavEncke predictor [6] operating with a fixed integration step of 120 s and modeling Newtonian (point source) gravity accelerations from the Earth, Sun, Moon, and Venus. When in Earth's gravitational sphere of influence, WeavEncke also models accelerations from Earth's excess equatorial mass (J_{20} spherical harmonic). Post-TVI velocity is differentially corrected using the precision trajectory's state transition matrix [7] to achieve desired boundary conditions, first at Venus flyby and finally at Earth return.

A. Earth Departure

The single-impulse TVI Δv computation performed in Section III as a crosscheck with MSC's mission summary value [1, p. 11, Table IV] is a simplification of the Earth departure MSC actually proposed, as described in detail by the following transcription [1, p. 3].

> The mission profile would consist of a single launch of a Saturn V into a circular orbit of about [+185 km] altitude followed by a coast to the required injection point. The S-IVB [Saturn V third stage] is then restarted and propels the [CSM+MM] into a high elliptical orbit. The period of the elliptical orbit is about 2 days. The apogee altitude of this orbit would be [+130,000 km]. … On the outbound leg of the elliptical orbit, a transposition and docking is performed [by the crew

aboard the CSM] and the MM is removed from the S-IVB. The crew enters the MM, deploys the solar panels, and checks out the vehicle. At apogee [about a day after launch], plane and perigee adjustments are made, if required. On the inbound leg, the crew returns to the Apollo CM and begins the countdown for interplanetary injection [TVI]. As perigee is approached [about 2 days after launch], the SM [Service Module] engines are started and the additional [0.9 km/s] injection velocity required for the Venus flyby is added.

This 3-impulse departure strategy achieves two objectives. First, it usefully expends the S-IVB's highly efficient cryogenic propellant promptly after launch, avoiding special design requirements to further reduce boil-off with respect to an Apollo lunar mission. Second, the strategy imposes a 2-day delay in departing for interplanetary space. During this delay, CSM+MM systems can be reconfigured and verified to be in readiness for a year away from Earth. These verified systems include the SM engines to be used for TVI and subsequent midcourse corrections targeting Venus flyby and Earth return. If a mission-critical system demonstrates insufficient integrity for interplanetary operations, a mission abort (particularly a time-critical one) is far easier to perform before TVI than afterward.

Because the MSC proposal contains only a limited amount of detail, multiple assumptions are necessary to carry out the Earth departure reproduction. These assumptions are summarized in the following itemized list.

- D01: launch is instantaneous from a location near Kennedy Space Center (KSC) at geocentric declination δ = 28.5° N; longitude λ = 80.6° W.
- D02: at launch, heading ψ = 90° (due east).
- D03: circular orbit coast from launch to S-IVB restart (reproduced as the prograde HA-1 impulse) is more than 1.5 orbits and less than 2.0 orbits in duration such that HA-1 and TVI have a northeasterly heading with $0 < \psi < 90°$.
- D04: at the apogee following HA-1, a prograde HA-2 impulse raises the subsequent perigee coinciding with the prograde TVI impulse to H = +400 km.

Precision Earth departure reproduction commences with TVI tentatively set to 4 April 1972 at 12:00:00 UT. Assumptions D01-D04, together with conic Leg 1 geocentric $\boldsymbol{v}_{\infty \boldsymbol{D}}$ on 4 April 1972, fix geocentric position and velocity immediately following the TVI impulse [8, p. 115, Fig. 6.17]. To illustrate Earth departure geometry, geocentric vectors and orbit planes are projected onto the Earth mean equator and equinox of epoch J2000.0 (J2K) celestial sphere whose radius is effectively infinite. This Figure 3 celestial sphere plot (CSP) is similar to a conventional map with longitude replaced by J2K right ascension in the horizontal direction, and latitude replaced by J2K declination in the vertical direction. Although north is still upward on a CSP, it should be noted east is to the left because the celestial sphere is being viewed from the geocenter looking outward at its *inside*. Consequently, CSM+MM prograde orbit motion through TVI is from right to left on a CSP.

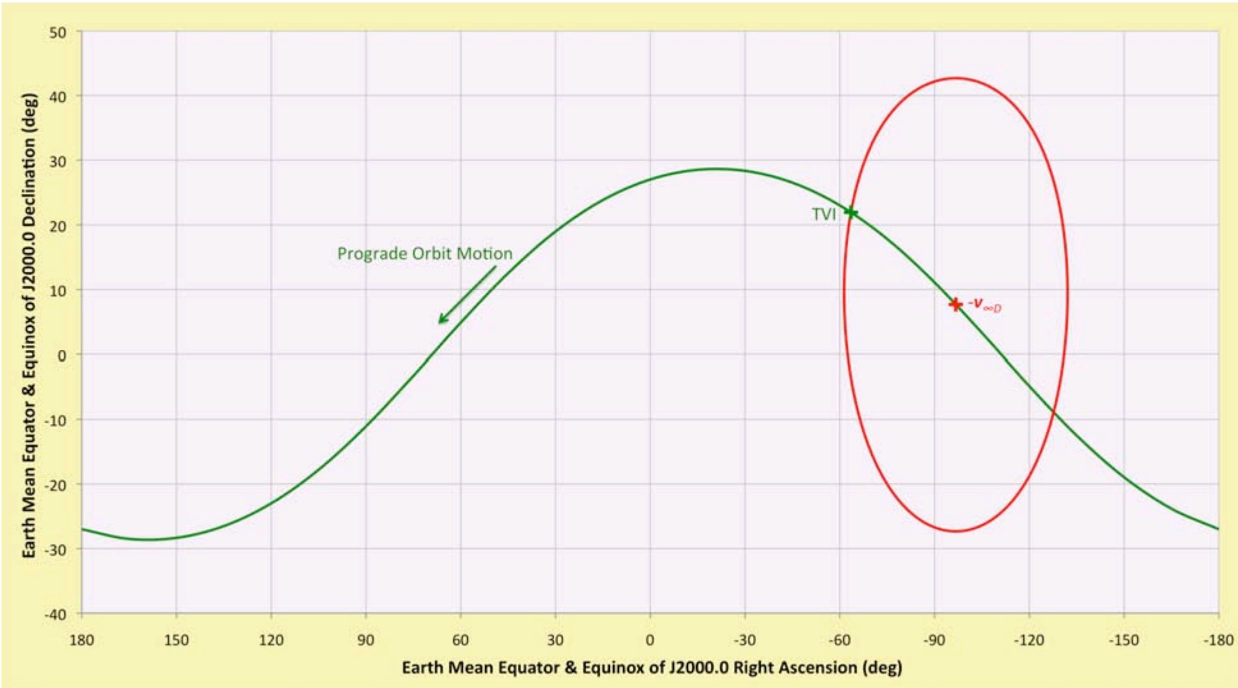

Figure 3. Earth departure geometry is illustrated on a celestial sphere plot (CSP). The red oval, actually a geocentric small circle, is the locus of all possible TVI points. This locus is centered on geocentric -$v_{\infty D}$ from the reproduction's conic Leg 1 trajectory whose heliocentric motion is illustrated in Figure 1. The green plane of orbit motion is unique because it is the only prograde great circle of its amplitude passing through -$v_{\infty D}$ immediately before the TVI locus is intercepted on a northeasterly velocity heading. In accord with foregoing narrative, orbit motion amplitude on the CSP is a product of Assumptions D01 and D02, while the heading at TVI is a product of Assumption D03.

From the 4 April 1972 at 12:00:00 UT TVI position, post-TVI velocity is scaled from a speed of 11.428 km/s to 10.567 km/s such that a pre-TVI geocentric conic period of exactly 2 days is imposed. Starting with this pre-TVI state, a series of 3 successive WeavEncke coasts backward in time is performed. The first coast extends from TVI back one day to the previous apogee. Velocity at this point is iteratively scaled to simulate the HA-2 impulse. Pre-HA-2 velocity scaling for each iteration is evaluated by a second one-day backward coast to the previous perigee such that H = +185 km at this point is ultimately achieved. Speed at the H = +185 km perigee is then scaled to achieve a nearly circular orbit with mean H = +185 km, thereby simulating the HA-1 impulse. The third backward coast is initialized with the pre-HA-1 state and extended more than 1.5 orbits until the first declination δ = 28.5° N is encountered in accord with Assumptions D01 and D03. This occurs at 2 April 1972 at 09:41:23 UT, the instantaneous launch time consistent with an initial TVI guess on 4 April 1972 at 12:00:00 UT.

But λ = 1.623° E following coast to 2 April 1972 at 09:41:23 UT, conflicting with Assumption D01's requirement that λ = 80.6° W at launch. This eastward bias in longitude relative to the desired KSC launch location is removed with 19,680 s of additional eastward Earth rotation by delaying TVI to 4 April 1972 at 17:28:00 UT. Retaining inertial TVI position and scaled pre-TVI inertial velocity at the delayed UT, a second series of three WeavEncke coasts backward in time is initiated. These coasts are terminated on 2 April 1972 at 15:09:18 UT when δ = 28.502° N; λ = 80.627° W are obtained and deemed to be sufficiently in accord with Assumption D01. At this point, the 3-impulse Earth departure trajectory reproduction is summarized in Table 3 and illustrated in Figure 4.

Table 3. Reproduced Earth departure events are summarized consistent with assumptions and analysis documented in this subsection's narrative. Although the UT of TVI is finalized, its Δv is preliminary and partly reflects a patched conic approximation to be refined in subsequent sections.

1972 UT Date & Time	Event
02 April @ 15:09:18	Instantaneous Earth launch from δ = 28.502° N; longitude λ = 80.627° W into a nearly circular orbit with average H = +185 km
02 April @ 17:56:16	S-IVB performs HA-1 impulse, raising apogee H to +120,188 km and imparting Δv = 2.951024 km/s
03 April @ 17:40:22	CSM performs HA-2 impulse, raising perigee H to +400 km and imparting Δv = 0.009179 km/s
04 April @ 17:28:00	CSM performs TVI impulse, departing Earth for Venus and imparting Δv = 0.860987 km/s

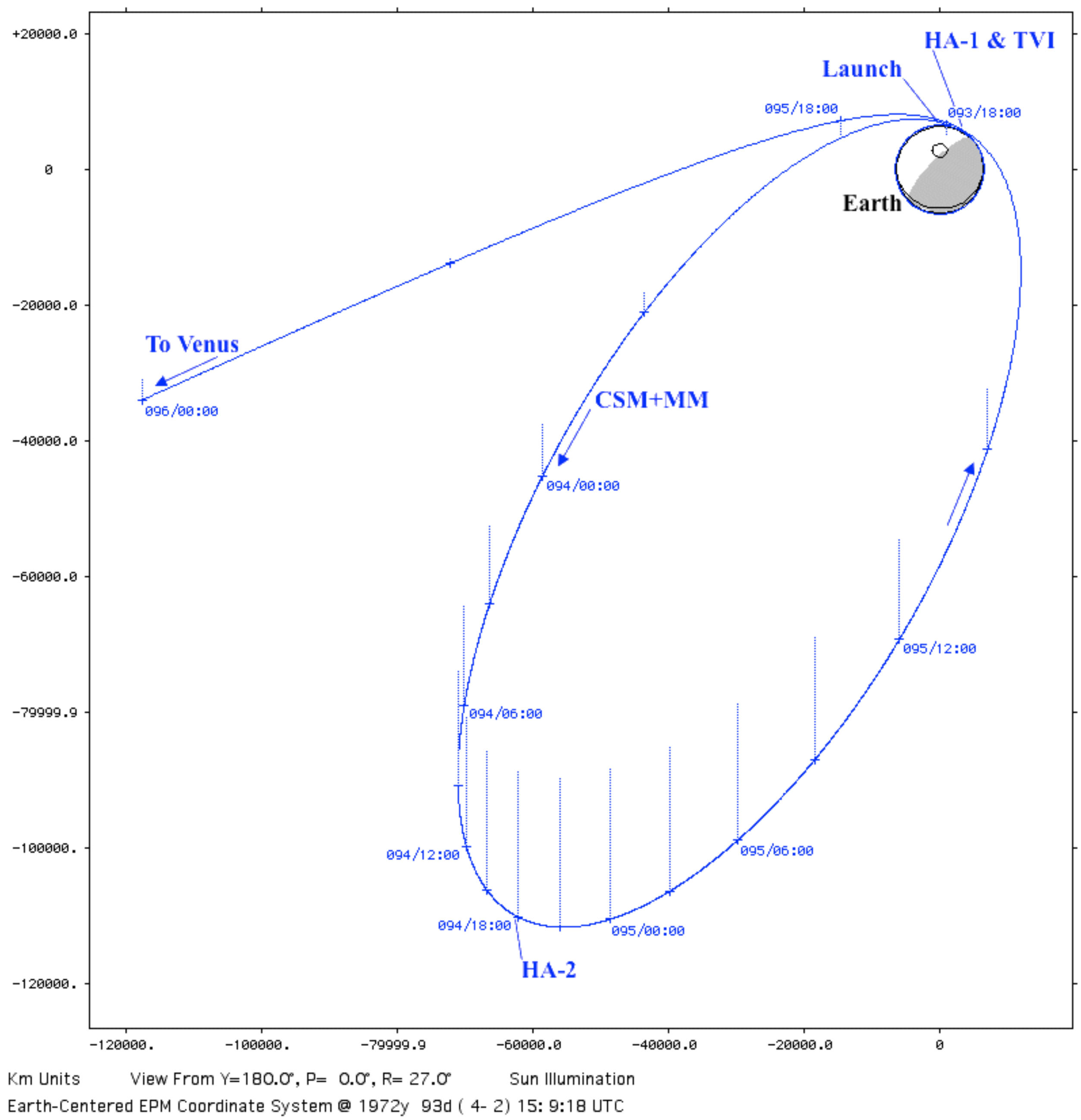

Figure 4. Geocentric CSM+MM motion is illustrated from instantaneous launch through TVI. Perspective is very nearly normal to the plane of motion. Time ticks (+ markers) appear every 3 hours and are annotated every 6 hours with UT in DOY/HH:MM format, where day-of-year (DOY) = 093 is 2 April in 1972. Dotted lines at time ticks are projections onto Earth's equatorial plane. The shaded area is Earth's nightside.

B. Preliminary Venus Flyby Targeting

Geocentric TVI position and UT computed in the previous subsection become fixed perturbed Lambert departure boundary conditions with which to differentially correct geocentric post-TVI velocity constrained by Assumptions D01, D02, D03, D04, and heliocentric conic Leg 1 $v_{\infty D}$. Almost any near-Venus position would serve as a reasonable arrival boundary condition with which to carry out initial post-TVI velocity differential correction. But heliocentric conic Leg 1 $v_{\infty A}$ and Leg 2 $v_{\infty D}$ impose geometric constraints permitting such a position to be determined in accord with a Venus flyby closely approximating one leading to Earth return. Consistent with Table 2, Venus arrival position is therefore fixed at periapsis with H = +550.3 km on 23 July 1972 at 00:00:00 UT.

A WeavEncke coast using the uncorrected post-TVI velocity misses the specified Venus arrival position by a distance of 270,421 km at the specified arrival UT. After 7 differential correction iterations, miss distance is reduced to 0.3 km, and post-TVI velocity is deemed sufficiently corrected. Total post-TVI velocity correction over the 7 iterations has a magnitude of 0.029773 km/s, indicating a precision trajectory solution highly consistent with the Leg 1 heliocentric conic approximation has been obtained.

The corrected post-TVI velocity is used to seed a second differential correction iteration in which arrival conditions are shifted to Earth return. Geocentric position at return is $-\boldsymbol{v}_{\infty A}$ from the Leg 2 heliocentric conic scaled to 200,000 km on 2 April 1973 at 00:00:00 UT. After 58 differential correction iterations, arrival position miss distance is reduced from 1,537,382 km to 0.5 km. This second set of iterations converges much more slowly than the first because highly nonlinear Venus flyby effects are encountered partway through a WeavEncke coast 3.3 times longer than Leg 1 alone (recall Leg 1 is a short way trajectory, while Leg 2 is a long way trajectory more than twice Leg 1's duration). Nevertheless, total post-TVI velocity correction over the 58 iterations has a magnitude of 0.024635 km/s. This correction moves Venus periapsis to H = +679.6 km on 23 July 1972 at 01:30:26 UT.

C. Earth Return

At the end of the previous subsection, a Venus flyby resulting in nearly head-on impact at Earth return had been targeted (return perigee H = -6295 km results primarily from an arrival position aligned with heliocentric conic Leg 2's $-\boldsymbol{v}_{\infty A}$). This return must be refined to produce a survivable atmospheric entry interface at low inertial flight path angle γ_{EI}. To initiate this process, the locus of possible perigee points (LPPP) is rendered on the Figure 5 CSP.

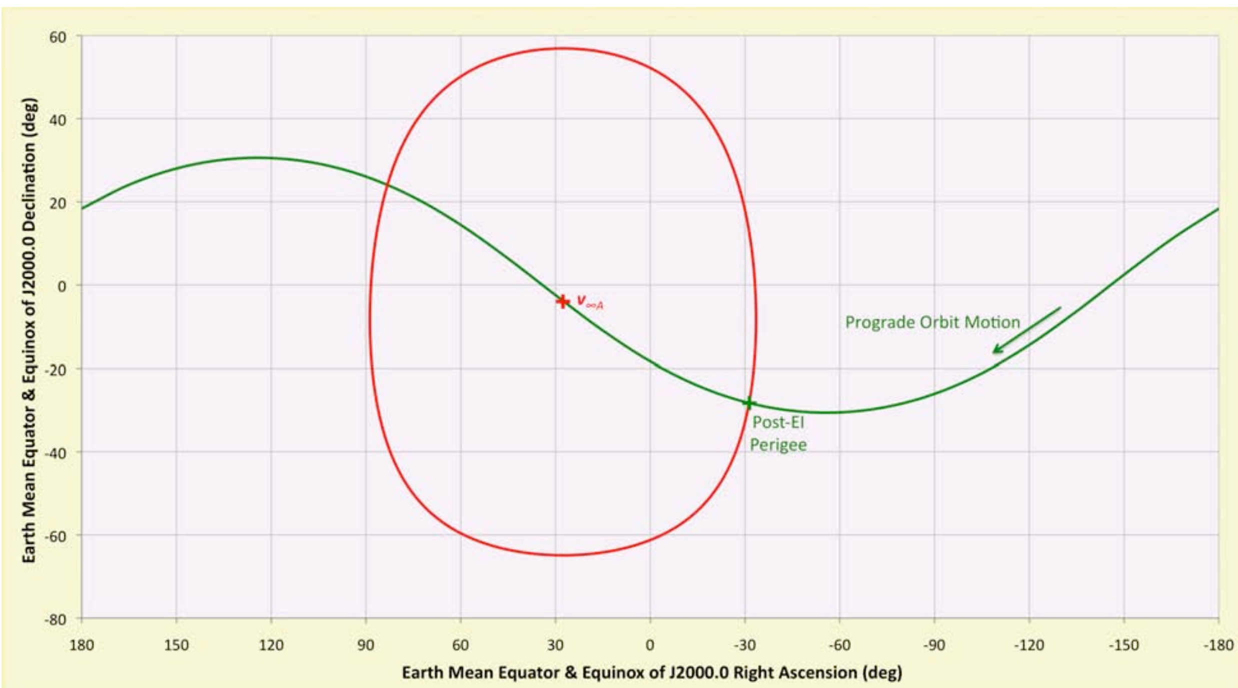

Figure 5. The Earth return locus of possible perigee points (LPPP) is plotted in red as a geocentric small circle on the J2K celestial sphere centered on heliocentric conic Leg 2's asymptotic arrival velocity $v_{\infty A}$ (red + marker). On this CSP, all prograde Earth returns move from right to left, first achieving atmospheric entry interface (EI), and then crossing the LPPP at perigee (green + marker) before passing through the $v_{\infty A}$ marker. The perigee marker's geocentric position is obtained from the fully corrected post-TVI CSM+MM state in Table 4 coasted one year back to Earth. It lies at δ = 28.399° S.

Visual inspection of Figure 5 leads to the first of multiple Earth return trajectory assumptions as follows.

R01: prograde Earth approach reaches perigee near δ = 30° S. This geometry imposes a northeasterly atmospheric entry ground track and tropical splashdown at low δ with minimal hypothermia concerns during crew water recovery. Prograde motion minimizes thermal and mechanical loads during atmospheric entry.

R02: in accord with Apollo lunar mission Earth return trajectory constraints, γ_{EI} is near -6.5°.

R03: entry interface occurs close to λ = 173° W, resulting in a Pacific Ocean splashdown near Hawaii.

The final differentially corrected Venus flyby solution reported in the previous subsection seeds a series of additional corrections to post-TVI velocity. Assumption R01, together with Leg 2's $\boldsymbol{v}_{\infty A}$ and specified Earth return perigee, determine the geocentric Earth return hyperbolic trajectory. The approach leg of this hyperbola is sampled for position at a geocentric distance of 200,000 km and assigned a preliminary epoch of 2 April 1973 at 00:00 UT. This position is the perturbed Lambert arrival boundary condition for further differential corrections. When these corrections achieve an arrival position miss distance of less than 1 km, WeavEncke coasts the corrected post-TVI state through Venus flyby to Earth return h = +121.92 km, where γ_{EI} is assessed. If γ_{EI} is below -6.5°, the geocentric Earth return hyperbola's perigee is raised; otherwise, perigee is lowered. Differential corrections to post-TVI velocity are then iterated to achieve miss distance less than 1 km with respect to the new hyperbola's approach position 200,000 km from Earth.

With the first series of Earth return differential corrections iterated to a reasonably converged γ_{EI} = -7.315°, entry interface λ = 31.286° E is computed. To satisfy Assumption R03 with minimal correction using Earth rotation, the perturbed Lambert arrival epoch at 200,000 km geocentric distance is shifted earlier to 1 April 1973 at 13:39 UT. Following further differential correction iterations, entry interface is achieved on 1 April 1973 at 19:57:56 UT with v_{EI} = 13.715 km/s (note the excellent agreement with Table 2's v_{EI} value for d_V = 23 July 1972), ψ = 83.300°, γ_{EI} = -6.532°, δ = -29.988°, and λ = 172.243° W. This result is considered to be sufficiently compliant with all Earth return assumptions. The associated geocentric J2K post-TVI state vector is provided in Table 4. It has Venus flyby periapsis on 22 July 1972 at 22:30:13 UT at H = +573.5 km (note the excellent agreement with Table 2's H Flyby value for d_V = 23 July 1972).

Table 4. Geocentric Cartesian J2K components of position *r* and velocity *v* are provided for the fully corrected post-TVI CSM+MM state coasting through Venus flyby and back to Earth.

Quantity	Value
UT Epoch	4 April 1972 @ 17:28:00
Geocentric J2K ***r*** (km)	+2802.386
	-5628.532
	+2531.677
Geocentric J2K ***v*** (km/s)	+9.070885
	+6.036874
	+3.449898

The vector difference magnitude between Table 4 velocity and pre-TVI velocity from the Earth Departure subsection gives TVI impulsive Δv = 0.862227 km/s, a value nearly identical to the preliminary TVI Δv appearing in Table 3. With the proposed budget for TVI impulsive Δv = 0.914 km/s [1, p. 5], the foregoing reproduced trajectory design appears practical in this context. The Venus-centered flyby trajectory is plotted in Figure 6. Note flyby periapsis is above the sunlit hemisphere of Venus as described in the MSC proposal [1, p. 4]. The Earth-centered return trajectory is plotted in Figure 7 and leads to a daytime splashdown. Both Figures 6 and 7 utilize time tick annotations with day-of-year (DOY) as a subfield. January 1 is DOY = 1, and December 31 is DOY = 366 in a leap year.

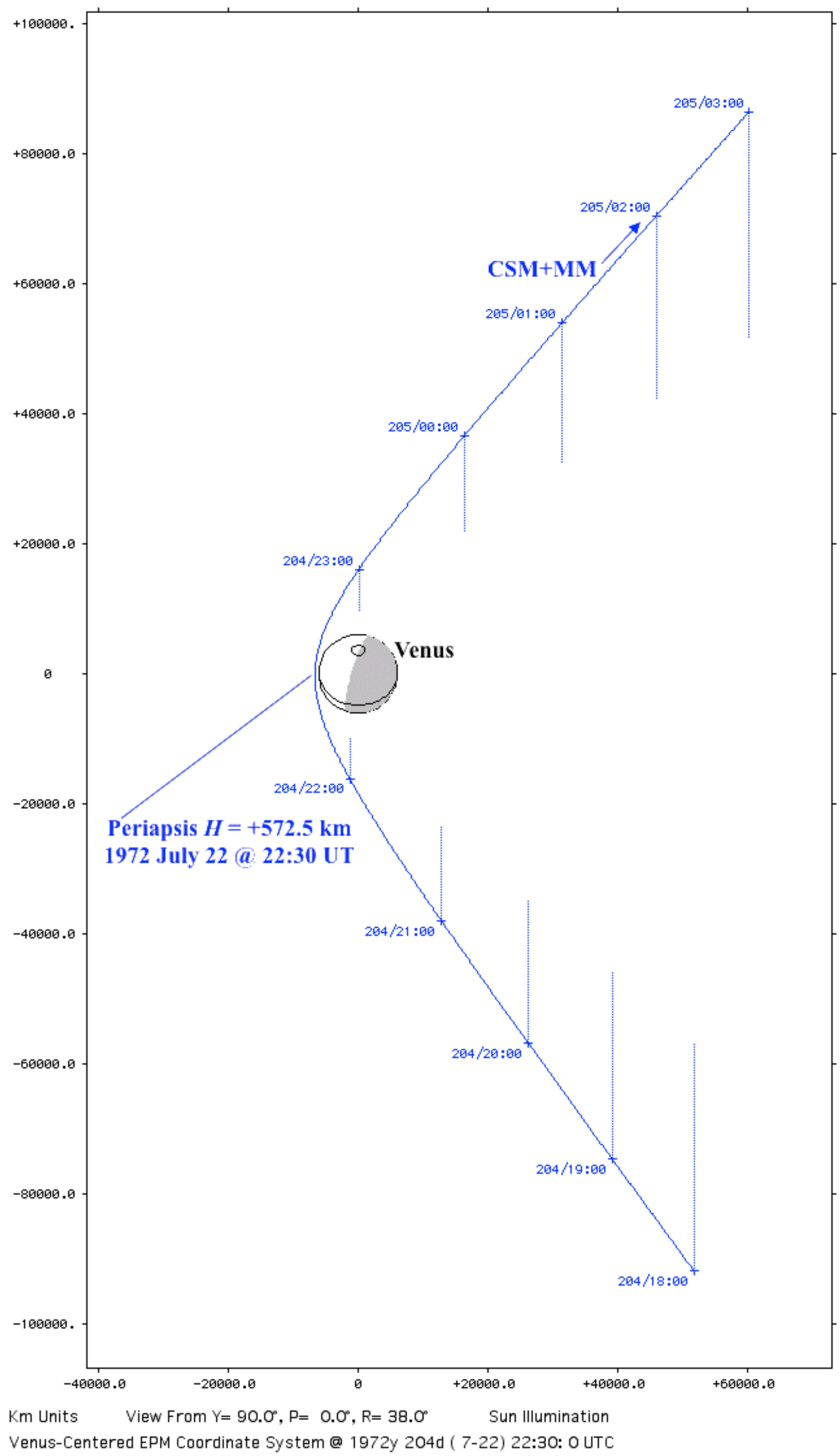

Figure 6. Inertial Venus-centered motion of the CSM+MM is plotted during flyby. Time ticks (+ markers) are at 1-hour intervals in DOY/hh:mm format. Dotted lines are projections onto the equatorial plane of Venus, and the shaded area is the nightside of Venus.

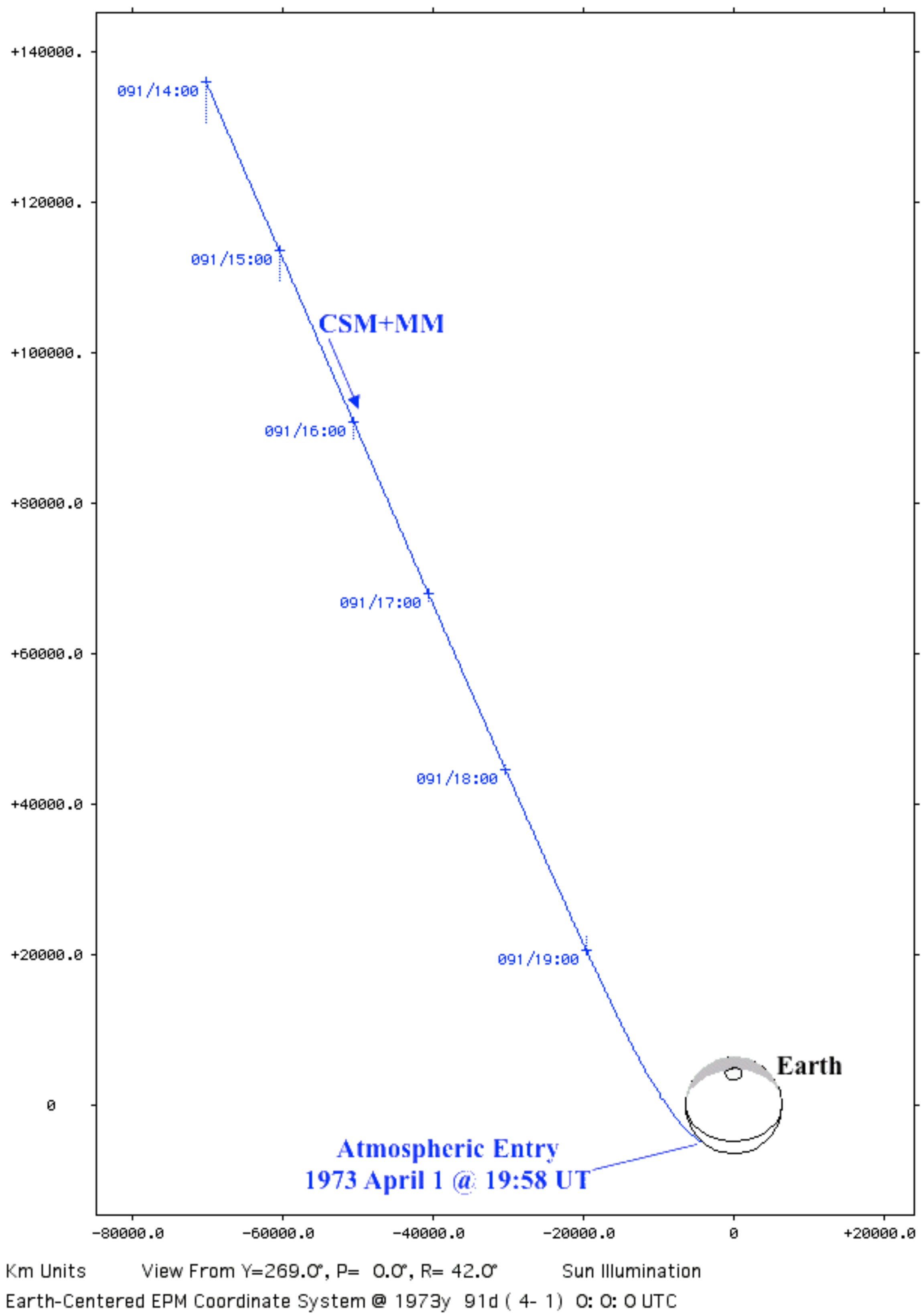

Figure 7. Inertial Earth-centered motion of the CSM+MM is plotted during return. Time ticks (+ markers) are at 1-hour intervals in DOY/hh:mm format. Dotted lines are projections onto Earth's equatorial plane, and the shaded area is Earth's nightside.

V. Conclusion

With respect to constraints documented in MSC's 1967 single-launch Venus flyby proposal, a reasonably compliant trajectory reproduction has been generated for TVI on 4 April 1972. The proposed maximum v_{EI} = 13.7 km/s is exceeded by the reproduction's 13.715 km/s, but a similar trajectory design in the proposal exhibits a speed violation 0.001 km/s greater (reference Table 1). A slight departure from the Table 1 MSC-proposed design is documented in Table 2 because the proposed Venus flyby on 22 July 1972 collides with Venus and must be delayed to avoid this fate. But the reproduction's precision trajectory design ends up converging on a Venus periapsis late in the UT day on 22 July 1972, rendering this departure insignificant.

The reproduction's precision trajectory solution is facilitated by a preliminary heliocentric conic trajectory using patched asymptotic conditions with respect to Earth departure, Venus flyby, and Earth return. These approximate conic trajectories are demonstrated to be remarkably accurate. Conic accuracy enables rapid assessment of Venus flyby conditions leading to Earth return. Accurate conic trajectories also seed differential correction iterations with initial guesses rapidly converging to precision results. These results are rendered in a high definition video of Earth departure, Venus flyby, and Earth return posted at https://youtu.be/3bzKsNkzP4s.

As the Introduction section observes, MSC's proposal is to some degree compromised by ethical issues arising from crew health risks juxtaposed with rewards from humans observing Venus at close range for a few hours. However, it is arguable this mission concept has considerably more integrity than recent proposals for human Mars flybys circa 2020.

VI. Acknowledgment

Author Gerald Brennan motivated this trajectory reproduction to help substantiate the technical basis for his alternate-history space exploration story titled *Island of Clouds*. Interested readers can acquire a copy of this story via http://www.tortoisebooks.com starting in early 2017. Thank-you, Jerry, for a fine collaborative effort!

References

[1] J. Funk and J. J. Taylor, "Preliminary Mission Study Of A Single-Launch Manned Venus Flyby With Extended Apollo Hardware", Mission Planning and Analysis Division, Manned Spacecraft Center, Houston, MSC Internal Note No. 67-FM-25, 13 February 1967.

[2] R.W. Orloff and D. M. Harland, *Apollo: The Definitive Sourcebook*, Springer-Praxis, 2006.

[3] L. F. Belew and E. Stuhlinger, *Skylab: A Guidebook*, NASA EP-107, U.S. Government Printing Office, 1973.[§§]

[4] D. R. Adamo and J. S. Logan, "*Aquarius*, A Reusable Water-Based Interplanetary Human Spaceflight Transport", 2014.[***]

[5] R. R. Bate, D. D. Mueller, and J. E. White, *Fundamentals of Astrodynamics*, Dover Publications, Inc., 1971.

[6] D. R. Adamo, "A Precision Orbit Predictor Optimized for Complex Trajectory Operations", *Astrodynamics 2003, Advances in the Astronautical Sciences*, Vol. 116, Univelt, San Diego, CA, 2003, pp. 2567-2586.

[7] D. R. Adamo, "Apollo 13 Trajectory Reconstruction via State Transition Matrices", *Journal of Guidance, Control, and Dynamics*, Vol. 31, No. 6, AIAA, 2008, pp. 1772-1781.

[8] C. D. Brown, *Spacecraft Mission Design*, AIAA Education Series, 1992.

[§§] This publication is available for download at http://history.nasa.gov/EP-107/contents.htm (accessed 17 January 2016).

[***] This publication is available for download at http://spaceenterpriseinstitute.org/2014/07/aquarius-a-reusable-water-based-interplanetary-human-spaceflight-transport/ (accessed 10 January 2016).