A NEW DISCOVERY: NONLINEAR INSTABILITY LEADING TO LOSS OF CONTROL OF AIRCRAFT

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Neglected elephants in the equations

- The original governing equations for aircraft roll ($\varphi$), pitch ($\theta$), and yaw ($\psi$) in the principal axis frame ($I_\varphi < I_\theta < I_\psi$):

$$I_\varphi \ddot{\varphi} + b_1 \dot{\varphi} + k_1 \varphi = (I_\theta - I_\psi) \dot{\vartheta} \dot{\psi} + M_x(t)$$
$$I_\theta \ddot{\theta} + b_2 \dot{\theta} + k_2 \theta = (I_\psi - I_\varphi) \dot{\varphi} \dot{\psi} + M_y(t)$$
$$I_\psi \ddot{\psi} + b_3 \dot{\psi} + k_3 \psi = (I_\varphi - I_\theta) \dot{\vartheta} \dot{\varphi} + M_z(t)$$

- The linearized equations:

$$I_x \ddot{\varphi} + b_1 \dot{\varphi} + k_1 \varphi = M_x(t)$$
$$I_y \ddot{\theta} + b_2 \dot{\theta} + k_2 \theta = M_y(t)$$
$$I_z \ddot{\psi} + b_3 \dot{\psi} + k_3 \psi = M_z(t)$$

$$ (I_y - I_z) \dot{\vartheta} \dot{\psi}, (I_z - I_x) \dot{\varphi} \dot{\psi}, (I_x - I_y) \dot{\vartheta} \dot{\varphi} - the inertial moments, similar to Coriolis force for hurricanes, should not be neglected
As nonlinear harmonic oscillation system:

\[
\begin{align*}
I_x \ddot{\phi} + b_1 \dot{\phi} + k_1 \phi &= (I_y - I_z) \dot{\theta} \dot{\psi} + M_x(t) \\
I_y \ddot{\theta} + b_2 \dot{\theta} + k_2 \theta &= (I_z - I_x) \dot{\phi} \dot{\psi} + M_y(t) \\
I_z \ddot{\psi} + b_3 \dot{\psi} + k_3 \psi &= (I_x - I_y) \dot{\phi} \dot{\theta} + M_z(t)
\end{align*}
\]

What matters the most is the exciting frequencies of these terms, not the amplitudes.
The most common maneuver in takeoff, cruise leveling and landing approach

The original governing equations:

\[ I_x \ddot{\phi} + b_1 \dot{\phi} + k_1 \phi = (I_y - I_z) \dot{\theta} \dot{\psi} \]
\[ I_y \ddot{\theta} + b_2 \dot{\theta} + k_2 \theta = (I_z - I_x) \dot{\phi} \dot{\psi} + M_{21} \cos(\omega_{21} t + \alpha_{21}) \]
\[ I_z \ddot{\psi} + b_3 \dot{\psi} + k_3 \psi = (I_x - I_y) \dot{\phi} \dot{\theta} \]

The current longitudinal dynamics (pitch control only):

\[ I_y \ddot{\theta} + b_2 \dot{\theta} + k_2 \theta = M_{21} \cos(\omega_{21} t + \alpha_{21}) \]
By the current practice, the aircraft response is (caterpillar mode)

\[ \varphi = 0 \]
\[ \theta = \theta_0(M_{21}) \cos(\omega_{21} t + \alpha_{21} + \xi_{21}) \]
\[ \psi = 0 \]

This pitch-only response is not always stable. Under certain conditions, the following roll, pitch and yaw bifurcation solutions (butterfly mode) exist.

\[ \varphi(t) = \sum_{i=1}^{\infty} \varphi_i(t) = \sum_{i=1}^{\infty} A_{1i} \cos(\omega_{1i} t + \beta_{1i}) \]
\[ \theta(t) = \sum_{j=1}^{\infty} \theta_j(t) = \sum_{j=1}^{\infty} A_{2j} \cos(\omega_{2j} t + \beta_{2j}) \]
\[ \psi(t) = \sum_{l=1}^{\infty} \psi_l(t) = \sum_{l=1}^{\infty} A_{3l} \cos(\omega_{3l} t + \beta_{3l}) \]
Amplitudes of the 1st modes

Solve the amplitudes:

\[ A_{11} = \frac{2}{\omega_{11}} \left\{ \frac{I_y I_z Z_{21} Z_{31}}{(I_z - I_x)(I_y - I_x)} \right\}^{1/2} \]

\[ A_{21} = \frac{2}{\omega_{21}} \left[ \frac{I_x I_z Z_{11} Z_{31}}{(I_z - I_y)(I_y - I_x)} \right]^{1/2} \]

\[ A_{31} = \frac{2}{\omega_{31}} \left\{ \frac{I_x I_y Z_{21} Z_{11}}{(I_z - I_y)(I_z - I_x)} \right\}^{1/2} \]

Roll impedance:
\[ Z_{11} = \sqrt{(\omega_{11}^2 - \omega_{10}^2)^2 + \left(\frac{b_1 \omega_{11}}{I_x}\right)^2} / \omega_{11} \]

Yaw impedance:
\[ Z_{31} = \sqrt{(\omega_{31}^2 - \omega_{30}^2)^2 + \left(\frac{b_3 \omega_{31}}{I_z}\right)^2} / \omega_{31} \]
**Pitch conditional instability criterion**

- Pitch unstable if

\[
M_{21} > 2I_yZ_{21} \left[ \frac{I_x I_z Z_{11} Z_{31}}{(I_z - I_y)(I_y - I_x)} \right]^{1/2}, \quad \text{pitch moment threshold}
\]

- Pitch stable if

\[
M_{21} \leq 2I_yZ_{21} \left[ \frac{I_x I_z Z_{11} Z_{31}}{(I_z - I_y)(I_y - I_x)} \right]^{1/2}
\]

- Proved in the book, if the first modes \( \varphi_1, \theta_1, \psi_1 \) exist, \( \varphi_2, \theta_2, \psi_2; \ \varphi_3, \ \theta_3, \ \psi_3, \ldots \) exist.
Pitch instability threshold verification

- A commercial aircraft model was used to do the numerical simulations.
- Pitch instability threshold in Scenario 1

The worst case: \( \omega_{21} = \omega_{10} + \omega_{30} \)

- Pitch instability threshold in Scenario 2

The worst case: \( \omega_{21} = \omega_{10} - \omega_{30} \)
The worst case – a resonance mode

- The pitch critical frequency:
  \[ \omega_{21} = \omega_{\text{critical}} \equiv \omega_{10} + \omega_{30} \]

- Pitch amplitude threshold
  \[ A_{PTH} \equiv \frac{2}{\omega_{10} + \omega_{30}} \left[ \frac{b_1 b_3}{(I_z - I_y)(I_y - I_x)} \right]^{1/2} \]

- Two dangerous situations,
  1. At stall,
     Roll damping: \( b_1 \rightarrow 0 \), then \( A_{PTH} \rightarrow 0 \)
  2. Yaw damper malfunction, turned off or hardover
     Yaw damping: \( b_3 \rightarrow 0 \), then \( A_{PTH} \rightarrow 0 \)
At the pitch critical frequency $\omega_{critical}$, aircraft is prone to nonlinear pitch instability before stall during takeoff.
Loss of control of aircraft

- A pitch control producing $18^\circ$ pitch oscillation at $\omega_{21} = 5.2 \ (Rad/s)$
- The pitch amplitude threshold: $A_{PTH} = 12^\circ$
- Pitch unstable: pitch $18^\circ > A_{PTH} = 12^\circ$
- The final steady state amplitudes depend on the external pitch control amplitude

“Uncommanded” roll $55^\circ$
“Uncommanded” pitch $37^\circ$
“Uncommanded” yaw $36^\circ$
Nonlinear pitch instability leading to Loss of control of aircraft on B737-236 Advanced G-BGJI

B737-236 mode change from caterpillar to butterfly happened 1.5 min before autopilot disconnect

Pitch amplitude about 0.5°

Pitch instability and inertial coupling effects acting alternately

$A_{PTH} = 0.35°$
Nonlinear pitch instability leading to Loss of control of aircraft on Ethiopian Airline Flight 302 B737 Max

General Overview of Flight

Flight 302 bifurcated from caterpillar to butterfly mode 4 seconds before stick shaker activated

Uncommanded roll began 4 seconds earlier

Stick shaker began 4 seconds later

Pitch $\approx 18^\circ >$ Boeing recommended pitch 15–16°

AOA $\approx 15^\circ$, critical AOA $\rightarrow b_1 \rightarrow 0, A_{PTH} \rightarrow 0$, pitch nonlinearly unstable
The list of nonlinear pitch instability leading to loss of control of aircraft during takeoff

- Northwest Flight 255 MD DC-9-82 crashed in 1987
- Delta Airlines Flight 1141 B727-232 crashed in 1988
- USAir Flight 405 Fokker F-28 crashed in 1992
- American Airline Flight 587 A300-605R crashed in 2001
- PT. Mandala Airlines Flight 091 B737-200 crashed in 2005
- Air Transat A310-308 C-GPAT upset in 2008
- Spainair Flight 5022 MD DC-9-82 crashed in 2008
- Gulfstream GVI (G650) N652GD test flight crashed in 2011
- Ethiopian Airlines Flight 302 B737 Max crashed in 2019
Nonlinear pitch instability demonstration
(www.youtube.com/watch?v=gG2-mu6I11A)

- Aircraft model with restoring and damping for roll and yaw
  \[ \omega_{10} = 2\pi, \quad T_{10} = 1 \text{ sec} \]
  \[ \omega_{30} = \pi, \quad T_{30} = 2 \text{ sec} \]

- The dangerous pitch frequencies
  Scenario 1: \( \omega_{21} = \omega_{10} + \omega_{30} = 3\pi \)
    \[ T_{21} = 0.7 \text{ s}, \quad A_{PTH-1} = \frac{2}{3\pi} \left[ \frac{b_1b_3}{(I_z-I_y)(I_y-I_x)} \right]^{1/2} = \frac{A_{PTH-2}}{3} \]
  Scenario 2: \( \omega_{21} = \omega_{10} - \omega_{30} = \pi \)
    \[ T_{21} = 2 \text{ s}, \quad A_{PTH-2} = \frac{2}{\pi} \left[ \frac{b_1b_3}{(I_z-I_y)(I_y-I_x)} \right]^{1/2} \]

- Experimental observation: \( A_{PTH-2} = 3A_{PTH-1} \)