This link leads to a Collings Foundation web page describing this amazing airplane.

As noted above, the French sister section of AIAA Houston Section is 3AF Midi-Pyrénées. The largest city in their territory is Toulouse France, the city mentioned in the name of this airplane, Toulouse Nuts.

I conclude the name Toulouse Nuts refers to the flights of this airplane in the Toulouse area of France in the World War 2 era. For English-speaking people, Toulouse Nuts certainly has meaning as Two Loose Nuts, a reference to the two-person crew of this 1944 pilot trainer airplane.

A loose nut (on a bolt) might cause mechanical problems during flight. A nut can refer to a crazy person, too; a friendly insult.